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A JOURNAL OF EXPERIMENTAL AND THEORETICAL PHYSICS

CONDUCTED
WITH THE CO-OPERATION OF THE

AMERICAN PHYSICAL SOCIETY

BY

EDWARD L. NICHOLS

ERNEST MERRITT, and FREDERICK BEDELL

CLXV

DECEMBER, 1909

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THE

PHYSICAL REVIEW.

ON THE ELECTRIC CHARGES ACQUIRED BY INSULATED POTASSIUM SALTS AND OTHER RADIOACTIVE SUBSTANCES IN HIGH VACUA.[1]

By J. C. McLennan.

SINCE the discovery in 1907 by N. R. Campbell and A. Wood[2] that potassium salts possess in a very definite though relatively small degree the property of radioactivity, the radiations from these salts have been examined by a number of investigators including among others Campbell,[3] McLennan and Kennedy[4] and Levin and Ruer.[5]

Amongst other properties of the rays examined by these investigators was their absorption by different thicknesses of various substances. From the results of such measurements the view came to be held that the rays from potassium salts were heterogeneous and consisted of several types varying in penetrating power from the beta rays of uranium downwards. The average intensity of the rays, moreover, was found to be about 1/1,000 of that of the radiation from uranium salts.

In a particular set of experiments which were carried out by Campbell the rays were passed between the plates of a large zinc grid and on emergence their intensity was ascertained from the conduc-

[1] Read before the Royal Society of Canada, May 26, 1909.
[2] Proc. Camb. Phil. Soc., Nos. 14–15, 1907.
[3] Proc. Camb. Phil. Soc., No. 14, 211, 1907.
[4] Phil. Mag., Sept., 1908.
[5] Phys. Zeit., 9 Jahr., No. 8, Seite 248.

tivity they imparted to a mass of gas in an ionization chamber. This grid was so arranged that alternate plates were metallically connected and consequently when the two sets of plates were joined to the two terminals of a battery an electric field was established in the intervals between the plates. Campbell found when a field of 8,000 volts was applied to this condenser that the intensity of the emerging rays was diminished by approximately 14 per cent. This led him to the conclusion that the rays consisted of streams of electrically charged particles, and from additional experiments on the direction of the deviation of the deflected beams he concluded that the charge carried by the rays was a negative one.

It followed therefore from these experiments that the rays in all probability belonged to the beta type of radiation.

In some absorption experiments made by the writer special care was taken to examine the rays from potassium salts for the presence of a radiation of the alpha type, but no evidence of the presence of such rays was obtained from the form of the experimental absorption-ionization curves, and the conclusion was drawn that if any such radiation was present, it must have been emitted with a velocity considerably less than that possessed by the alpha rays from other well known radioactive substances.

More recently E. Henriot[1] made an examination of the rays emitted by potassium salts, and while the results of his experiments agree in the main with those of the earlier investigators, they differ from them in that they point to a complete homogeneity in the radiation. For on measuring the ionization produced by the rays after passing through various layers of tinfoil, he found on plotting the logarithms of the ionization current for the rays from potassium sulphate and potassium chloride against the thicknesses of tinfoil used that the points all lay on a straight line.

Henriot, in his paper on the subject, also claims that in the course of his measurements he found some slight evidence of the presence of slightly penetrating rays of the alpha type in the radiation from the potassium salts examined.

Quite recently an examination was made by Strutt[2] of the

[1] Henriot, Comptes Rendus, 148, pp. 910–912, 1909.

[2] Strutt, Proc. Roy. Soc., Series A, Vol. 81, No. A547, p. 278.

amount of helium present in a number of saline minerals, and the results of this examination which are given in Table I. make it clear that the potassium-bearing minerals and especially sylvine are rather remarkable for the comparatively great amount of helium which they have been shown to contain. In discussing the origin of the gas Strutt considered that it was altogether improbable that

TABLE I.

Mineral.	Composition.	Helium c.mm. per 100 Grammes.
Rock salt	$NaCl$	0.0233
Sylvine	KCl	0.55
Carnallite	$KMgCl_2 6H_2O$	0.151
Kieserite	$MgSO_4 H_2O$	0.0179

the minute traces of uranium and radium present in sylvine could account for so much helium, and in view of Campbell and Wood's discovery of the radioactivity of potassium salts he was led to regard potassium itself as the source of the helium in the saline minerals examined.

In view of the identity established by Rutherford between the alpha particles and the atoms of helium, it would follow from the view taken by Strutt — that the origin of helium in the saline minerals is the potassium which they contain — that rays of the alpha rays should be present in the radiation from potassium salts.

With the exception of Henriot no one, however, seems to have observed any indication of the presence of such rays. Usually however these rays have been detected and identified either by their fluorescent and ionizing action or by the deflections which they undergo in electric or magnetic fields. But for these methods to be applicable the alpha particles must have velocities which will take them away a few millimeters at least from the substances which emit them.

It is quite possible then that potassium and even other substances may be emitting alpha particles with exceedingly low velocities, and that these rays have hitherto escaped detection largely through the inadequacy of the means adopted to bring them into evidence.

In looking for a means of demonstrating the possible existence of these rays it seemed to the writer worth while to make an attempt to investigate the radiations emitted by the potassium salts, by ex-

amining them for the acquisition of an electric charge when placed on insulating supports within a highly exhausted vessel.

Amongst others M. and Mme. Curie,[1] Paschen,[2] Strutt,[3] Aschkinas[4] and Makower[5] have applied this method with success to the investigation of different types of radiation. In Paschen's experiments it was found when a lead cylinder containing a small quantity of a radium salt was insulated in a vessel from which the air was removed that the lead cylinder acquired a positive charge through the action of the rays which were emitted. This effect was thought by Paschen to prove that the gamma rays from radium consisted of streams of rapidly moving negatively charged particles but this was afterwards shown by Eve[6] to be due to the excitation and consequent emission of a secondary radiation of the beta type in the lead by the passage through it of the gamma rays.

The method too was applied by Strutt (*loc. cit.*) in his interesting experiment popularly known as the radium clock, and more recently it was applied by Aschkinas to demonstrate the existence of the delta rays from deposits of polonium, to which radiation attention was first drawn by the experiments of Sir J. J. Thomson,[7] Logeman,[8] Ewers,[9] and others.

Makower has also recently used it to measure the beta radiation from radium and from these measurements to deduce the number of beta particles emitted per gram of that substance.

By applying this method to the radiation from potassium salts the writer has observed in certain cases that potassium salts when insulated in high vacua, acquire a positive charge, but up to the present it has not been found possible from the characteristics of the acquired charge to decide whether this charge arises from the radiation being wholly of the beta type or whether it is due to the emission of rays of both the alpha and beta types with the beta type of radiation in excess.

[1] M. & Mme. Curie, Comp. Rend., CXXX., p. 647, 1900.
[2] Paschen, Wied. Ann., 14, 1, pp. 164–171, 1904.
[3] Strutt, Phil. Mag., Nov., 1903.
[4] Aschkinas, Phys. Zeit., 8, p. 773, Oct. 24, 1907.
[5] Makower, Phil. Mag., Jan., 1909.
[6] Eve, Nature, Sept. 8, 1904.
[7] Thomson, Camb. Phil. Soc. Proc., 13, pp. 49–54, Feb. 18, 1905.
[8] Logeman, Proc. Roy. Soc., Series A, Vol. 78, No. A523, p. 212.
[9] Ewen, Phys. Zeit., March, 1906, pp. 148–152.

Among other phenomena observed when applying the method are those associated with the Volta effect. If a metallic rod or vessel connected to an electrometer be placed within and insulated from a second metallic vessel, it will be found when both the containing vessel and the insulated vessel are joined to earth, that generally a potential difference will exist between them. This potential difference will of course be more marked when the two vessels are made of different metals. It will also, however, be observable ordinarily when the two vessels are made of metals supposedly the same, but in this case the existence of the effect only goes to show that some slight difference exists in the composition or condition of the two pieces of metal used in the construction of the two vessels.

If now, with the arrangement just described, where one vessel is placed within and insulated from a second, the earth connection of the electrometer be removed it will be found that the insulated body or vessel more or less rapidly acquires an electrical charge, the sign of the charge being determined by the direction of the potential gradient. This charge arises from the air or gas between the two vessels possessing a conductivity which is imparted to it either by radiations emitted from the walls of the containing and contained vessels or from the passage through the gas of the penetrating radiation which is known to be present at the surface of the earth. The conductivity possessed by the air will tend to diminish the potential difference set up between the two bodies when they were both joined to earth, and the charge acquired by the electrometer under the action of the conductivity when earth connection to the inner vessel is removed, will give a measure of the magnitude of the Volta effect.

This then is one type of charge which is nearly always acquired by such an insulated system as that described. It has, however, certain definite characteristics which make it readily detectable. For example, the rapidity with which this charge is acquired depends largely on the degree of conductivity possessed by the intervening gas, and as this conductivity can be increased at will by bringing more or less near to the vessel a small quantity of radium or other radioactive body, it is possible to make the insulated system practically take up at once a charge which will suffice to annul the Volta effect.

If, further, now, the inner vessel or body in the arrangement described possesses in addition a radiating surface which emits alpha or beta rays, the charging action of these rays can generally be brought into evidence by a reduction of the pressure of the gas between the two bodies. This reduction of the pressure will produce a diminution in the conductivity of the gas but it will not affect the rate of emission of charged particles from the radiating surface. Consequently as the pressure of the gas is reduced the insulated system should, under the action of its charged radiation, acquire a charge, and so set up a potential difference between the inner and outer vessels.

The actual current through a gas, however, at any particular pressure depends so long as the saturation current has not been attained, upon the potential difference producing the current, and so it happens that at each pressure a state of equilibrium is brought about, by virtue of which the insulated system is maintained at such a potential as will produce a current through the gas of such magnitude as to exactly counterbalance the gain of charge through the emission of the radiation.

But as the pressure of the gas is lowered the equilibrium potential of the insulated system becomes gradually greater and greater, and the sign of the charge on the free system corresponding to this gradually increasing potential difference will be the opposite of that of the emitted radiation which is in excess.

Further, the extent of the equilibrium potential corresponding to any particular pressure will give a measure of the magnitude of this excess radiation.

When applying the method to the investigation of any particular radiation, the earth connection to the insulated system should first be broken and time allowed for the free system to come into equilibrium under the action of the conduction current arising from the Volta effect. When this equilibrium has been reached the scale reading corresponding to the position then assumed by the movable system may therefore be taken as the initial reading in considering the charging action due to the radiation itself. Two lines of procedure are then open. If the radiation from the insulated system be a strong one the movable system of the electrometer will take

up the equilibrium position practically at once for any particular pressure and so the pressure may be lowered by stages and the corresponding scale readings observed. But if the radiation be a weak one it may take the movable system a long time to acquire the equilibrium potential corresponding to any particular pressure and under these circumstances it is best, if the object of the investigation is to ascertain the character of the excess emitted radiation, to reduce the pressure as rapidly and as low as possible, and then, while maintaining the low pressure, to observe whether a movement occurs in the movable system of the electrometer.

If such a motion occurs, the direction of the motion will indicate the sign of the charge acquired and the rate of movement will give a measure of the magnitude of the intensity of the charge producing radiation.

II. Charging Action of the Radiation from Polonium.

Some preliminary attempts were made to obtain a charging effect with some potassium salts in a high vacuum but it was soon found that such charging action was exceedingly small, and it was thought best to carry out a few parallel experiments with some of the better known radioactive substances, in order to gain some information regarding the pressures at which a charging action would be exhibited by various types of rays from active substances placed in a number of differently shaped vessels.

The first experiment was made with polonium deposited on a strip of copper. The area of this deposit was about 6 sq. cm. The copper strip which carried it was supported by a piece of amber insulation at the center of an-air tight brass cylinder 20 cm. long, and about 5 cm. in diameter. This cylinder was connected to a McLeod gauge and also to a Gaede mercury exhausting air pump. The insulated copper strip bearing the polonium was also connected to the free quadrants of a Dolzaleck electrometer, which gave about 200 mm. divisions deflection per volt. With this arrangement it was found repeatedly when the earth connection to the free quadrants was broken, that the needle moved slightly in the positive direction and came to rest about three centimeters from the zero reading. This deflection was taken as a measure of the Volta effect.

As the pressure was lowered no further change occurred in the electrometer reading until a pressure of approximately .4 mm. was reached. At this pressure the quadrants always commenced to gain a positive charge and as the pressure was still further lowered the needle at once moved out and took up a definite position corresponding to each pressure. When the pressure was taken below .01 mm. the charging action became very marked and produced deflections beyond the limits of the scale.

A set of readings taken with this polonium-coated copper strip in the neighborhood of the initial rise due to the radiation is given in Fig. 1, and a curve drawn from the readings is shown in the same

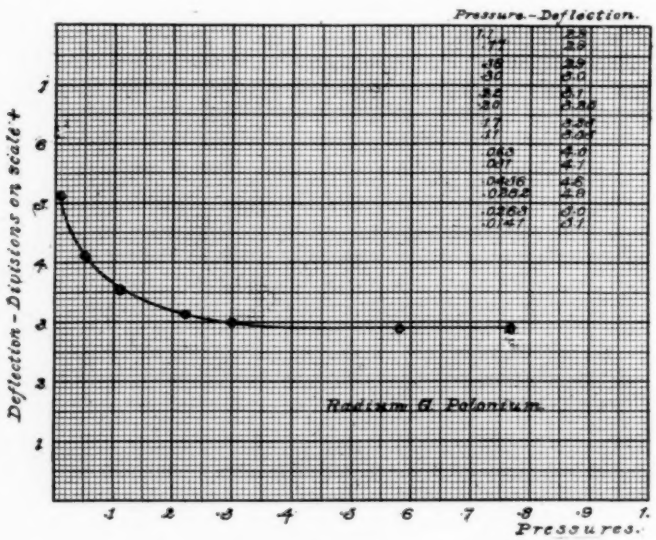

Fig. 1.

figure. This curve is typical of the different ones obtained with this active product.

It shows quite clearly that this substance omits an excess of negatively charged particles, and it also exhibits very definitely the pressure at which this excess comes into evidence.

III. Charging Action of Secondary Rays Excited in Aluminium by the Gamma Rays from Radium.

In this experiment one milligram of radium bromide was enclosed in a small sealed glass tube, whose walls were about 2 mm. thick. This glass tube was placed inside a tube of aluminium, which was then closed at the top and bottom. This tube which was about 7 cm. in length and had walls about 2 mm. in thickness was then insulated as before and suspended in the brass cylinder used in the last experiment. In this experiment the charging action was very much more marked than in the experiments with polonium. It also exhibited certain characteristics which were not observed in the measurements with that substance.

A set of results which illustrate its behavior is shown in Fig. 2.

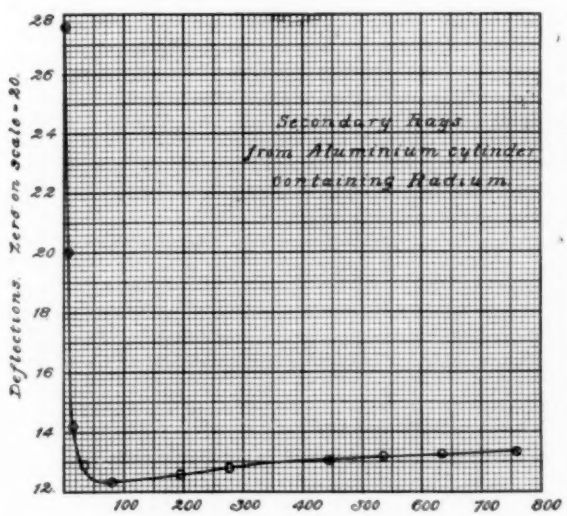

Fig. 2.

When the outside vessel and the free system were joined to earth the zero reading on the scale was 20, and as soon as the earth connection to the quadrants was removed the needle moved out at once in the negative direction and took up a position at 13.3. When the pressure was lowered the negative charge on the quadrants gradually increased at first but ultimately at a pressure of about 80 mm. this acquisition of a negative charge ceased and when the pressure was

reduced, still further the charging action was reversed and for lower pressures the free system more and more rapidly gained a positive charge. In this experiment the Volta effect would make the aluminium positive to the outside brass cylinder and consequently this would explain the acquisition of an initial negative charge when the earth connection to the free quadrants was broken. It is probable, however, that the difference between 20 and 13.3 did not represent the exact magnitude of the Volta effect for in this case the radiations present would include secondary rays from the walls of the brass outside cylinder as well as others from the walls of the inner aluminium tube. The reading 13.3 would then represent the equilibrium potential acquired by the free system through the agency of (1) the Volta effect, (2) the conduction current, (3) the secondary

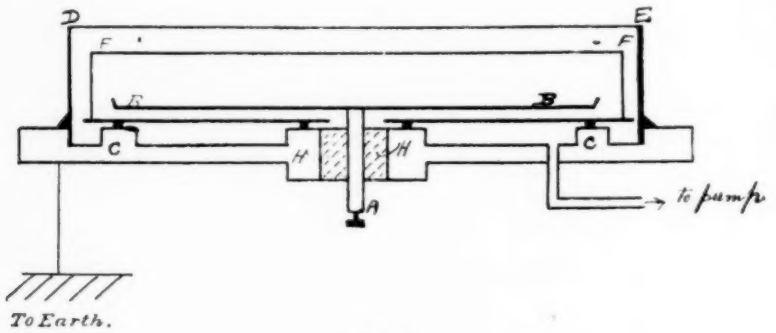

Fig. 3.

rays from the aluminium tube, (4) the secondary rays from the brass outside vessel and also (5) any tertiary rays from the two opposing surfaces which might be present. The initial increase in the negative charge acquired by the aluminium tube in the range of pressures extending to 80 mm. was probably due to modifications produced by the lowering of the pressure in the amount of secondary rays coming to the aluminium tube from the walls of the brass outside vessel. The charging action of these would initially mask the charging action of the various excited secondary rays issuing from the surface of the aluminium tube. But ultimately, as the pressure was lowered a point would be reached when these secondary rays from the outside vessel would exert their maximum

effect. For lower pressures the conduction current through the gas would become less and less. This would bring the secondary rays from the aluminium tube more into evidence and so account for the rapid increase in the positive charge acquired by the free system, which the curve shows took place at the lowest pressures. From this experiment it will be seen that the charging action of the secondary rays produced in aluminium by the gamma rays from radium was brought into evidence at a very much higher pressure, 80 mm., than a similar action by the rays from polonium.

IV. On the Charges Acquired by Uranium Salts at Low Pressures.

In this experiment the form of the vessel used in making the examination is shown in Fig. 3. A flat tray of brass *BB* was supported by an ebonite plug, on the brass plate *CC* shown in the figure. A shallow brass cover *FF* was placed over the tray *BB* and rested on the lower plate. Over the whole a second cover *DE* was placed which fitted snugly into a groove provided in the lower plate *CC*. In making the experiment the salt to be examined was placed on the tray *BB*, the covers were then placed in position, and finally all the joints made air tight with sealing wax. The salt used in making the experiment was a sample of uranium nitrate. The measuring instrument used was again a quadrant electrometer, but in this case it possessed a sensibility of 600 divisions per volt. In carrying out the experiment it was found that when the earth connection to the free quadrants was broken, the needle moved out in the positive direction on the scale for a distance which represented approximately one fortieth of a volt, and came to rest in this position. This deflection was taken to represent the deviation arising from the Volta effect. The chamber was then slowly evacuated, but as the exhaustion proceeded no additional deflection of the needle was observed. An attempt was then made to reduce the pressure as low as possible but although a pressure of .003 mm. was reached no indication was obtained of any additional charge being acquired by the insulated tray which carried the uranium nitrate.

As it is known that both alpha and beta rays are emitted by these salts, this result pointed to the conclusion that the rays carried

away from the salt equal amounts of positive and negative electrical charges. In order to test this matter still further a second form of exposing chamber was constructed.

Its design is shown in Fig. 4. The salt was spread out in thin layers some 3 or 4 mm. thick on a set of 12 brass trays *AA* whose diameters were about 9 cm. These were all fastened to a central brass rod *C* which was suspended in the outer brass cylinder by an insulating support of ebonite. This vessel carried a small tray of P_2O_5 in the bottom and it was also provided with a set of plates,

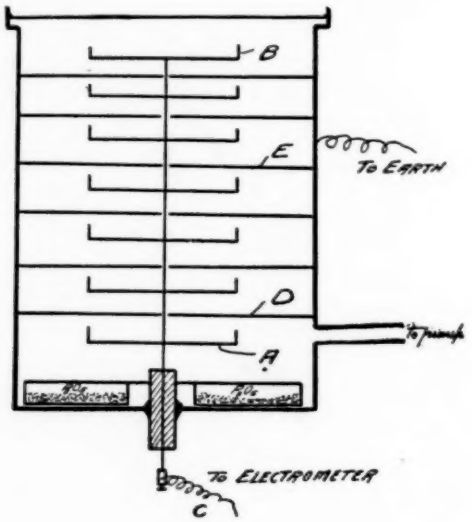

Fig. 4.

DD, so arranged between the trays as to intercept the rays issuing from the salt. The apparatus was provided with a cover as shown in the figure, and all the joints were again made air tight with solder and sealing wax.

With this form of apparatus the Volta effect was again in the positive direction, but the corresponding displacement was greater in this case, and represented approximately one quarter of a volt. After the deflection resulting from the Volta effect had become steady, the air was rapidly exhausted from the receiver and although the pressure was reduced to .02 mm., no indication of any charging action was obtained.

From the fact that no charging action was obtained in either of the experiments with the two forms of apparatus, it seems clear that the alpha and beta rays from the nitrate of uranium at least carry away with them from the salt equal amounts of positive and negative electricity.

In order to test the accuracy of this conclusion an additional set of experiments was made with uranium nitrate when using this second form of apparatus. The salt on each tray was covered with two layers of aluminium leaf each about .007 mm. in thickness. As this thickness was sufficient to cut off all the alpha rays,[1] but only a small proportion of the beta rays it was evident that a charging action should occur, and some experiments were made to see at what pressure it could be observed.

As aluminium in the Volta series is positive to brass, the charge acquired in this case through the annulling of the Volta effect was a negative one. On reducing the pressure it was found that the electrometer needle commenced to move away from the stationary position assumed under the influence of the Volta effect, when a pressure of approximately 8 cm. of mercury was reached, and the direction of displacement was such as to correspond to the acquisition of a positive charge. The sign of the charge acquired, it will be seen, fitted in then with what was to be expected from the absorption of the rays. For the higher pressures the rate of motion of the needle as it moved to take up the deflection corresponding to a selected pressure was slow and on this account the second line of procedure mentioned above was followed. In taking the observations the pressure was reduced as low as possible and then the rate of motion of the needle noted as it moved out to take up the equilibrium position.

Examples of these movements of the needle are shown in Figs. 5 and 6. In the one case the movement corresponds to a pressure of .15 mm. and in the second to a pressure of .08 mm. The rate of movement, as was to be expected, is greater for the lower pressure than it was for the higher one. The fact that positive charges were acquired under these circumstances lends further support to the conclusion drawn from the earlier experiments that equal quantities

[1] Phys. Zeit., 8, p. 773, Oct. 24, 1907.

of opposite kinds of electricity were emitted with the rays from uranium nitrate when freely exposed.

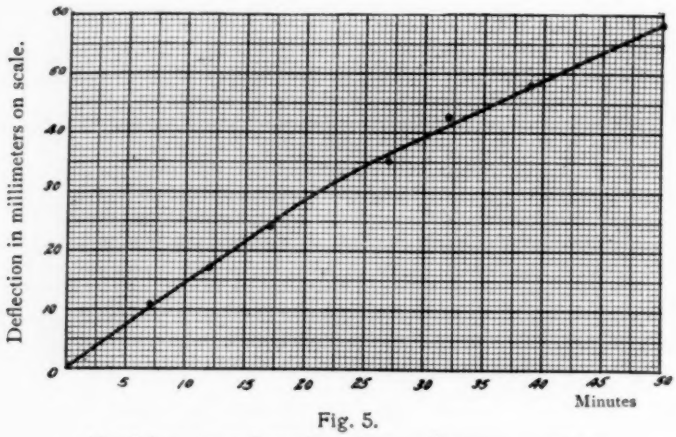

Fig. 5.
Aluminium-covered uranium nitrate. Pressure = .15 mm.
Voltage on needle = 243 positive.

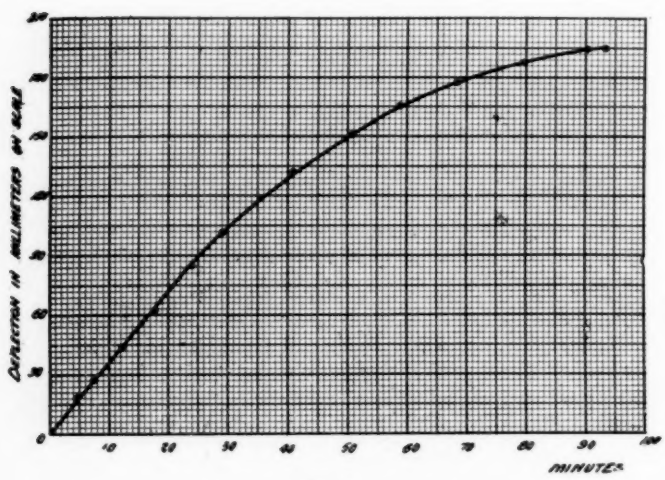

Fig. 6.
Aluminium-covered uranium nitrate. Pressure = .08 mm.

V. POTASSIUM SALTS: ELECTRICAL CHARGING ACTION.

In making the observations with potassium salts pieces of apparatus similar in form and dimensions to the two shown in Figs. 3 and 4 were used in turn. In the first experiment a layer of potas-

sium chloride, about 4 mm. thick, was spread on the tray *BB* in Fig. 3, and then the space surrounding the tray was as highly evacuated as possible.

Some preliminary observations had shown that extreme care had to be taken to see that no electrification was imparted by friction to the insulating support *HH*, during the operation of setting up the apparatus. Even touching it slightly with the finger or allowing its surface to be scraped with any object, was found to leave an electrification behind which gradually disappeared, but which in doing so brought irregularities into the results and often vitiated them.

In taking the readings given below, exceptional care was therefore taken to make certain that all the insulating supports were entirely free from electrification. The electrometer was also exceptionally well set up, the needle being so carefully adjusted and the suspension wire kept so free from torsional strain that when both pairs of quadrants were joined to earth no displacement from the zero took place for hours.

With the apparatus in this condition the test was made for a charging effect with the potassium chloride, first when the needle of the electrometer was positively charged to 240 volts, and secondly when it was negatively charged to the same voltage. The sensibility of the electrometer was such that a potential difference of 1 volt between the quadrants gave a deflection of 600 mm., divisions on the scale. In making the test the pressure was lowered to .003 mm., and maintained at this low pressure throughout the measurements.

The results obtained with the needle positively charged and also when it was negatively charged are shown in Fig. 7. From these curves it will be seen that in both cases the tray containing the salt slowly acquired a positive charge. The rate at which this charge was acquired, moreover, was greater when the needle was positively charged than when it was negatively charged, but this difference was probably due to the small conduction current which must have existed in the air between the needle of the electrometer and the free quadrants.

The effect it will be seen is a small one and the time required to make the readings was long. The readings, however, were quite

regular, and with the precautions and care taken there seems no reason for interpreting the results otherwise than as representing a charging action produced by the radiation emitted by the insulated potassium salt. Observations were made under precisely the same conditions with the tray empty, but these failed to show any such charging action as that obtained with the salts. These experiments

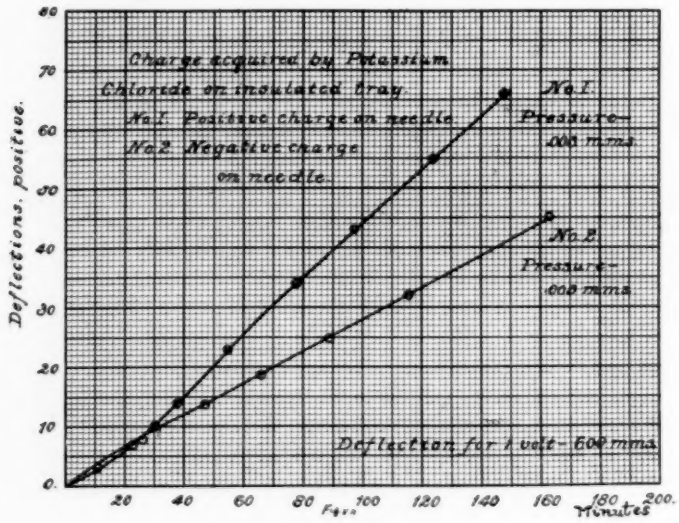

Fig. 7.

with the potassium chloride were made, as stated above, with the apparatus shown in Fig. 3, but they were carried out before the apparatus was used in the uranium nitrate experiments, and consequently there was no possibility of a connection between the charging action observed with the potassium salt and any contamination of the apparatus through contact with the uranium nitrate. This charging action with a salt of potassium was also observed with the apparatus shown in Fig. 4. In this set of observations the salt used was the sulphate of potassium. Two sets of measurements were again made. In the one the potential on the needle was positive and equal to 243 volts, and the pressure of the residual air in the apparatus was .08 mm. In the second set the needle was kept at a negative potential of 243 volts, and the pressure of the air was maintained at .06 mm. In these two latter measurements it was

not found possible to lower the pressure of the gas below the readings stated, although the Gaede pump used in the exhaustion was kept in action continuously. It is possible that the vessel containing the salt had some minute opening in its walls, but although careful search was made for such an opening none was detected. The readings taken in these observations are plotted in Figs. 8 and 9.

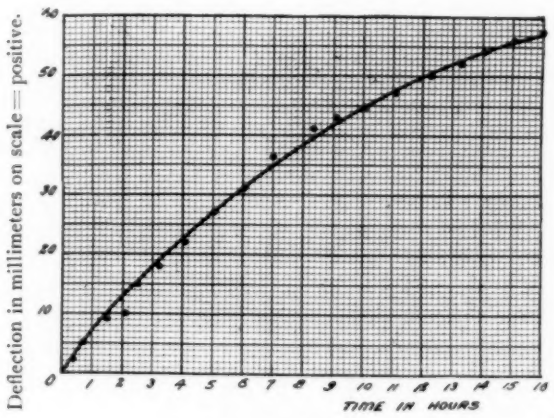

Fig. 8.
Charging action of potassium sulphate. Pressure = .08 mm.
Voltage on needle = 243 positive.

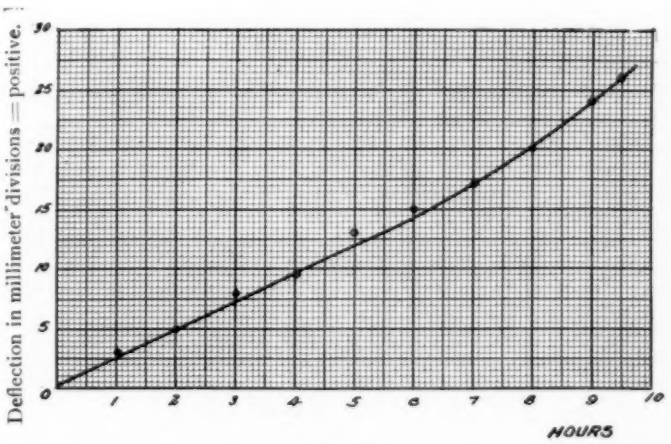

Fig. 9.
Charging action of potassium sulphate. Pressure = .06 mm.
Voltage on needle = 243 negative.

Here again it will be seen the charging action was a positive one in both cases and when the needle was positively charged its motion was somewhat more rapid than when it was negatively charged. In both of these two latter sets of observations the Volta effect produced a deflection in the negative direction when the earth connection to the free quadrants was broken at atmospheric pressure; but in each case after these earth connections were broken the complete apparatus was left undisturbed for some 8 hours in order that the needle might take up its stationary position corresponding to the Volta effect displacement. From time to time during this period observations on the scale readings were made and in this way the time was ascertained when the deflections became stationary. When these stationary states had been reached the exhaustions were then made and when the limiting pressures given above were reached, readings on the charging effect were commenced. These, it will be seen, confirm the results obtained with the potassium chloride in the earlier experiments.

These results show that in high vacua both potassium nitrate and potassium sulphate emit an excess of charged particles of the beta type. The effect observed in all cases was small and it will be noted that it required exceptional conditions to bring it into evidence. From the results obtained so far it has not been found possible to decide whether any radiation of the alpha type is emitted by the potassium salts or not. It is the intention of the writer, however, to examine the charging effect with the potassium salts covered with thin layers of aluminium. The use of such layers while preventing the emission of any alpha rays which might be present should not diminish to any great extent the intensity of the beta radiation.

The measurements made in this investigation demanded considerable patience and care, and I am indebted to Mr. M. E. Liezert and Mr. B. L. Cooke for kind assistance on occasions during the progress of the observations.

PHYSICAL LABORATORY,
TORONTO, May 26, 1909.

THE HEAT OF DILUTION OF ZINC AMALGAMS.

By W. D. Henderson.

WHEN the solution of a salt of a given concentration is reduced to a lower concentration by the addition of water, heat may be absorbed or evolved. Similarly, when an amalgam is diluted by the addition of mercury a heat change occurs. The heat thus absorbed or evolved is called the heat of dilution.

The relation of the heat of dilution to the electromotive force of a simple concentration cell has been pointed out by Carhart,[1] and his conclusions verified experimentally by Mellencamp for the case of a series of salts in solution, the heats of dilution of which had been determined calorimetrically by Thomsen.

The original object of the present research was to extend the investigation, above noted, to the determination of the heats of dilution of zinc amalgams. The method employed was to set up a cell of the type, conc. Zn amal. : sol. $ZnSO_4$: dil. Zn amal., and to measure the factors E.M.F. and dE/dt, and then from these data to compute the heat of dilution by means of the Helmholtz equation,

$$E = \frac{H}{nF} + T\frac{dE}{dt},$$

in which

E = electromotive force in volts,
T = absolute temperature,
dE/dt = change of E.M.F. per degree centigrade,
n = valence of the positive ion,
F = 96,500 coulombs per gram equivalent, and
H = heat of dilution in joules per gram ion.

Electrolyte.

The electrolyte used, both in the preparation of the amalgams and in the concentration cells as finally set up, was a nearly saturated solution of zinc sulphate, made from pure recrystallized salt.

[1] Trans. Am. El. Chem. Soc., XII., 399, 1907.

AMALGAMS.

The amalgams were prepared electrolytically out of contact of air. A short rod of pure zinc, carefully wrapped in filter paper, was used as the anode; while pure twice-distilled mercury served as the cathode. In the preparation of the amalgams for each experiment, two cells were set up, one cell containing 20 grams of mercury, the other 40 grams. After thoroughly exhausting by means of an air pump, the cells were filled with hydrogen, after which they were connected up with a silver coulometer, the three being in series, and a current of 0.3 ampere passed through the system for a given time. At the close of the experiment, the mass of silver deposited in the platinum cup of the coulometer was carefully weighed, and from this it was possible to determine very accurately the amount of zinc deposited in each amalgam cell. The electrochemical equivalent taken for silver was 0.0011182 gram per coulomb.

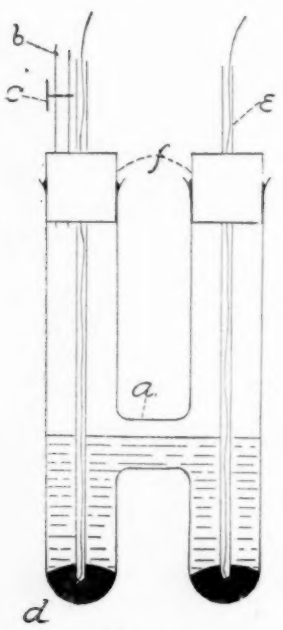

Fig. 1.

CONCENTRATION CELL.

The cell used for the determination of E.M.F. and dE/dt was of the H-form shown in Fig. 1.

The cross-arm *a* was made relatively short (about 3 cm.) in order to reduce the resistance as low as possible. Moreover, the diameter of this arm was made rather large, so that when the cell was set up, there remained a space above the liquid in the arm, thus making it possible to exhaust both limbs by means of a single connection at *b*, which was provided with a stop-cock *c*. Contact with the mercury electrodes *d* was made by means of platinum wires sealed into glass leads *e*. Each limb of the cell was closed at *f* with rubber stoppers, thoroughly cleaned and sealed in by means of a thin solution of Le-Page's glue. At the beginning of the work an all-glass cell was

employed. It was found, however, that this precaution was not necessary with amalgams of the concentrations employed.

Set Up of the Cell.

In setting up the cell, the first step was to test the platinum tips, which communicated with the amalgams, with respect to an initial E.M.F., no tips being employed that were not practically "dead" to each other. This precaution was necessary in order to make it possible to measure the E.M.F. to the sixth decimal place. The required amount of zinc sulphate was then added to the cell, after which the amalgams were transferred from the cells in which they had been prepared. The platinum leads were put in place, and the cell was immediately connected up with a powerful air pump and exhausted. In the tube connecting the air pump and the cell was a three-way stop-cock which allowed communication both with the pump and also with an apparatus for the generation of pure hydrogen. A few strokes of the pump were sufficient to reduce the pressure in the cell to the boiling point of the solution at room temperature. The cell was first exhausted, then filled with hydrogen, this process being repeated a number of times. After being thoroughly washed out with hydrogen, the cell was filled with this gas and then sealed off. It was then placed in a constant temperature bath and readings taken.

E.M.F.

E.M.F. determinations were measured against a Weston Normall cell by means of a Wolff potentiometer reading directly to 0.00001 volt. The resistance of the concentration cell was so low that it was easily possible to interpolate readings to the sixth decimal place.

dE/dt.

The change of electromotive force with change of temperature for a given pair of amalgams was obtained by determining the E.M.F. for two or more different temperatures, and then computing the change of E.M.F. per degree centigrade. Four baths were employed having respectively temperatures of 0°, 20°, 25° and 30°. The ice-bath could be used, however, only for the lower concentrations, for reasons which appear later.

The method employed was, in general, to place the cell first in the 25-degree bath; then to carry it through the entire range of temperatures possible for the given solution and amalgam, allowing the E.M.F. readings to come to a constant value in each case. Usually each cell was taken through several such cycles, the time required being two or three days.

In every case the dE/dt values were positive, that is, an increase of temperature gave on increase of electromotive force.

CONCENTRATION OF THE AMALGAMS.

In the preparation of each set of amalgams the mercury used in every case was in the ratio of 1 : 2 by mass — that is, 20 grams of mercury in one leg of the cell, and 40 grams in the other, the same mass of zinc being deposited in each. In speaking of the concentration of any pair of amalgams, then, reference is always made to the ratio between the mass of zinc and mercury of the more concentrated electrode. For example, if in 20 grams of mercury (the amount used in every case for the concentrated electrode) there was deposited 0.2 gram of zinc, this amalgam would be referred to as a concentration of 1 per cent.

When the amalgams are prepared electrolytically and in series, as in the present case, and it is desired to plot curves in which the concentration is one of the variables, the above method and notation make for both accuracy and convenience.

DATA.

Concentration cells set up as described gave remarkably constant and consistent values, no evidence of oxidation effects being manifested over a period of several days.

The following record illustrates very well the constancy of the values obtained.

Even with a concentration as low as 0.5 per cent., the values of the E.M.F. readings show practically no deviation, for a given temperature, as shown by Table II.

For convenience in referring to the different concentrations, the entire series is divided into three groups — namely, group one comprising the lower concentrations; group two, the higher concentra-

TABLE I.

Cell II-B. Concentration, 0.918 *per cent.*

Date.	Notes.	Temp.	E.M.F.
November 16, 9:00 A. M.	Cell in 25° bath.		
" 4:00 P. M.	"	25°	0.007966
" 4:15 "	Cell in ice bath.		
" 4:30 "	"	0°	7243
" 5:00 "	"	0°	7240
" 5:15 "	Cell in 25° bath.		
" 6:45 "	"	25.03	7970
" 7:00 "	"	25.03	7970
" 7:15 "	Cell in ice bath.		
" 7:30 "	"	0°	7240
" 7:45 "	"	0°	7240
" 8:00 "	"	0°	7240
" 8:15 "	Cell in 25° bath.		
" 17, 10:00 A. M.	"	25.01	7970
" 1:00 P. M.	"	25.02	7970

tions; and group three, those concentrations in the 2 per cent. region.

For the lower concentrations — that is, from 0.5 to about 2 per cent. — the electromotive force came to a constant value within a few hours after the cell was set up and placed in the 25-degree bath. Moreover, an increase of concentration gave a uniform decrease of E.M.F. and a slight though uniform increase of dE/dt values.

TABLE II.

Cell IX-C. Concentration, 0.496 *per cent.*

Date.	Notes.	Temp.	E.M.F.
March 22, 2:00 P. M.	Cell in 25° bath.		
" 6:00 "	"	25.04	0.008383
" 9:00 "	"	25.04	8388
"	Cell successively in baths at 30, 20 and 0 degrees.		
" 23, 10:00 A. M.	Cell in 25° bath.		
" 1:30 P. M.	"	25.05	8385
" 2:30 "	"	25.03	8380
"	Cell in baths at 30 and 0 degrees.		
" 7:00 P. M.	Cell in 25° bath.		
" 24, 8:00 A. M.	"	25.05	8387

In the region of the higher values — from about 2.5 to 3 per cent. — the behavior of the amalgams with respect to E.M.F. and dE/dt was quite similar to that of group one, with the exception of an increase of the slope of the E.M.F. and dE/dt curves, as shown in Fig. 2.

Concentrations lying in the 2 per cent. region gave anomalous results, and require special consideration. Three things are to be noted with respect to this region: First, the electromotive force of

TABLE III.

Cell XIII–C. Concentration, 2.148 *per cent.*

Date.	Notes.	Temp.	E.M.F.
March 30, 4:00 P. M.	Cell in 25° bath.		
" 5:00 "	"	25.05	0.006757
" 9:00 "	"	25.06	6758
"	Cell in 30° bath.		
" 31, 8:00 A. M	Cell in 25° bath.		
" 9:00 "	"	25.04	6833
" 10:00 "	"	25.04	6833
"	Cell in 30° bath.		
" 6:00 P. M.	Cell in 25° bath.		
April 1, 8:00 A. M.	"	25.04	7110
" 10:00 "	"	25.04	7110
"	Cell successively in baths at 30 and 20 degrees.		
" 8:00 P. M.	Cell in 25° bath.		
" 2, 7:45 A. M.	"	25.12	7113
"	Cell in baths at 30 and 20 degrees.		
" 6:00 P. M.	Cell in 25° bath.		
" 3, 8:00 A. M.	"	25.05	7113
" 2:00 P. M.	"	25.05	7110
"	Cell in ice bath. E.M.F. constantly shifting.		
" 3:00 "	Cell in 25° bath.		
" 5:00 "	"	25.05	7115

a given set of amalgams did not come at once to a final constant value, but seemed to approach constancy by steps; second, the E.M.F. curve makes a sharp change in direction in this region; and third, the dE/dt values undergo a sudden and enormous increase.

The first point noted above is well illustrated by Table III., which record may be taken as a type of the behavior of the cells of the 2

per cent. region. This cell (No. XIII–C, concentration 2.148 per cent.) was under test for something over 90 hours, and approached a constant value (E.M.F.) by at least two distinct steps. The cell was first placed in the 25-degree bath, where it gave a constant E.M.F. reading of 0.006757 volt. It was next thoroughly agitated and transferred to the 30-degree bath, at which temperature readings were taken, after which it was brought back to the 25-degree bath, where the E.M.F. registered was 0.006833 volt. On again being agitated and transferred to the 30-degree bath, and then again brought back to the 25-degree bath, the cell came to a final constant value of 0.007110 volt, after which no series of changes to either higher or lower temperatures seemed to affect its constancy.

In computing the heats of dilution H, the final constant E.M.F. values for 25 degrees were substituted for E in the equation,

$$E = \frac{H}{nF} + T\frac{dE}{dt}.$$

A summary of the data obtained from a series of fourteen sets of concentrations with respect to E.M.F., dE/dt and H values is given in Table IV.

TABLE IV.

No.	Conc. of Amal.	E.M.F.	dE/dt	H
1	0.499 per cent.	0.008360	0.0000280	+ 3
2	0.960	7890	281	− 193
3	1.180	7675	309	− 296
4	1.512	7365	320	− 419
5	1.676	7120	307	− 390
6	1.842	7000	300	− 375
7	1.928	6855	277	− 270
8	2.040	6840	263	− 200
9	2.148	7110	318	− 457
10	2.224	6867	400	− 976
11	2.305	6320	900	−3960
12	2.412	5730	1710	−8736
13	2.740	4562	1800	−9480
14	3.070	3410	1830	−9920

The relationship of the four sets of values of Table IV. is beautifully shown by the following curves of Fig. 2:

DISCUSSION OF FIG. 2.

The three curves of Fig. 2 are plotted, with respect to abscissas, in terms of concentrations; with respect to ordinates, in terms of E.M.F., dE/dt and H respectively. Concentrations, to repeat, refer to the ratios of zinc to mercury of the more concentrated amalgam of each cell. The E.M.F. and dE/dt curves, plotted above the heavy line, represent observed values and are positive; the H-curve represents computed values, which are negative. The entire H-curve is not given in the figure, since the range of its values is so great, as compared with those of the other two curves.

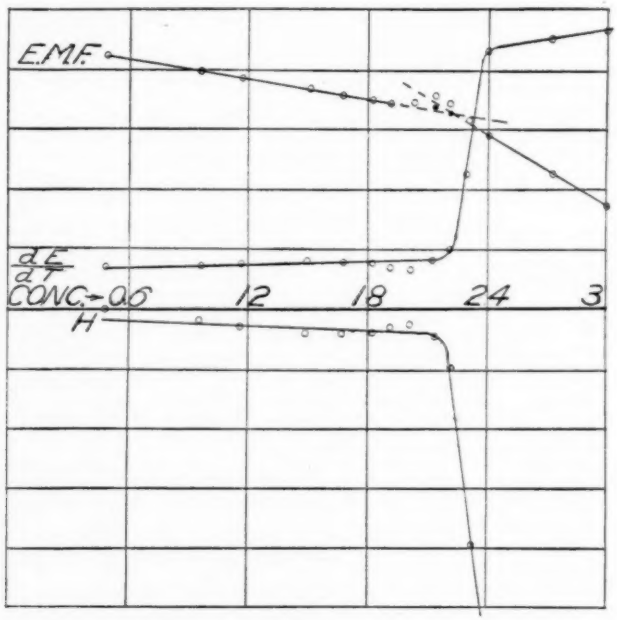

Fig. 2.

The E.M.F. curve is the most significant one of the group. It consists, as shown in Fig. 2, of two straight lines intersecting in the 2 per cent. region. The light circles represent final constant values; the dark points, initial constant values (see Table III.). This curve is worthy of special note for several reasons: First, while it is very uniform for the lower and the higher concentrations, it shows a decided break in the 2 per cent. region. This break in the E.M.F.

curve, together with the simultaneous and remarkable changes indicated by the dE/dt and H-curves, undoubtedly indicates the occurrence of some fundamental change in the constitution of the amalgam, the form of the curves of this region suggesting something analogous to a transition point. Second, the dark points of the E.M.F. curve, which represent *initial* constant values for the given concentrations, lie, it will be observed, very nearly on the projections of the two straight lines. This would imply that in this region the amalgams tend initially to obey the same laws which govern the formation of the straight line portions of the curve. And in conclusion, the final constant values of the 2 per cent. region are higher than the initial values, and the process by which they are reached does not appear to be a reversible one.

Summary.

1. For the case of a concentration cell as herein specified, it was found that an increase of the zinc content of the amalgams composing the electrodes gave a decrease of E.M.F.

2. A similar increase of the zinc gave an increasing value for the dE/dt factor.

3. Assuming that the Helmholtz equation applies throughout the range of the concentrations used, it was computed that the heat of dilution for the entire series of amalgams is negative, and varies from a value that is nearly or quite negligible for a concentration of 0.5 per cent., to about 10,000 joules per gram ion for a concentration of 3 per cent.

4. In the 2 per cent. region a marked change occurs in the constitution of the amalgam.

5. A further and more thorough investigation of the behavior of zinc amalgams in the 2 per cent. region is desirable. Such an investigation is now in progress by the writer of this paper.

In conclusion, it is desired to state that this research was undertaken at the suggestion of Prof. H. S. Carhart, whose counsel throughout the progress of the work has materially aided in bringing the investigation to its present state.

University of Michigan,
May, 1909.

THE SECULAR SOFTENING OF HARD STEEL.

BY C. BARUS.

1. *Introductory.* — The interesting result recently obtained by Miss L. C. Brant, showing persistent and marked softening of a batch of nineteen hard wires, at ordinary temperatures in the lapse of a second period of twelve years, has recalled to my mind a number of older observations to which it may be worth while to refer. The wires in question were tempered 24 years ago, by aid of a special electrical contrivance, devised by myself and used in the old Clifton Laboratory of the Johns Hopkins University, to which I was given access for the purpose through the courtesy of Professor H. A. Rowland. The strong heating currents available made it easy to temper several hundred steel wires glass hard, and to a high degree of uniformity, as the tests showed, throughout their length. In fact the hardness reached was exceptional (specific resistance 45 microhms) and much above the maximum obtained in earlier experiments abroad (specific resistance 40 microhms), by somewhat similar methods. The bearing of this on the phenomenon of temper will appear below, § 5.

The chief result gained in the present paper is this, that the limits of temper corresponding to any temperature, however practically useful and sufficient, are temporary limits only. All hard steel tends to become ultimately soft, at a rate that has been sharply recognized at all temperatures, at least in comparative values.

2. *Temper as Related to the Temperature of the Annealing Bath and the Time of Exposure to it.* — In order to interpret the results for secular softening it will be necessary to make use of the data[1] investigated by Doctor V. Strouhal and myself for the general behavior of tempered steel.

They contain an admirable example of the law subject to which

[1] Bulletin 14, U. S. Geological Survey, 1885; or Wied. Ann. der Physik, XI., pp. 930–977, 1880.

chemical reaction proceeds in case of a solid, in its dependence on time and temperature, particularly so, since every detail of the variation may be traced to a nicety. These results have been systematized in the following table.

TABLE I.

Change of Specific Resistance, s (microhm, c.cm., 0° C.) of Glass-hard Steel Rods, with the Temperature T and the Time t (minutes) of Tempering.

T	t	Observed. s	Computed. s	T	t	Observed. s	Computed. s
66°	0m	40.8	40.8	185°	0m	39.1	39.1
	60	39.4	39.4		10	26.7	26.8
	120	38.8	38.8		30	25.4	25.6
	180	38.3	38.3		60	24.7	24.8
					120	24.3	24.0
100°	0m	39.0	39.1		180	23.7	23.5
	10	36.4	35.2				
	30	34.5	34.0	330°	0m	37.7	37.7
	60	33.2	33.1		1	17.2	17.9
	120	31.8	31.9		30	16.8	16.7
	180	31.1	31.3		90	16.7	16.3
				1,000°	—	15.0	

Here T denotes the temperature at which the glass-hard rod is tempered by exposure during a time t minutes, s the resulting specific resistance at zero Centigrade. The computed values of s are discussed at the end of this and the subsequent paragraphs.

From the appearance of these curves (see figure) and from general considerations one would suppose that an equation like $-dn/dt = an$, where n is the number of hard molecules per c.cm. and a a constant for any given temperature T, would express the conditions of decay met with. It would then follow that the sum of the conductivities per c.cm. is

$$\frac{1}{s} = \frac{1}{s_s} - \left(\frac{1}{s_s} - \frac{1}{s_h}\right) e^{-at}$$

where s denotes the specific resistance of the sample, s_h and s_s the initial final hard and soft specific resistances, respectively, and t minutes is the time of exposure to the given temperature T. I made these computations from the observations at the initial parts

of the curves using $a/2.3$ for common logarithms,

$T =$	66°	100°	185°	330°,
$10^6 \times a/2.3 =$	356	1,926	7,490	29,560;

and also for the two ends of the curves by introducing an intermediate constant, or special hard state for each rod. But it is easily seen that the equation is inapplicable. In fact one need merely compare a criterion like the quantity $\log [(1/s_s - 1/s_h)/(1/s_s - 1/s)]$ with t, which relation should give a straight line on the assumption, to see that the result is about as curvilinear as the original equation. The only advantage of the a-values (first series) is that they convey some clue as to the character of these variations with temperature, T.

The next supposition as to the law of decay would be $-du/dt = au^2$ where the hard molecules are supposed to react on each other. This is equally inapplicable, by the same tests.

It is clear therefore, that the law of decay contained in the above results, Table I., is not apt to be of a simple character. I therefore contented myself with the most direct exponential capable of reproducing the data in question, with a reasonable degree of approach. This may be found by beginning with the curves for resistance and deducing the law of decay from it. A comparison of the above criterion and t^n, where n is the fraction, shows that

$$\frac{1}{s} = \frac{1}{s_s} - \left(\frac{1}{s_s} - \frac{1}{s_h}\right) e^{-at^{30}/T} \qquad (1)$$

where T is the temperature at which the rods are tempered and t the time of exposure, gives a fairly good account of the phenomenon as a whole, so far as these observations are concerned. The constants are

$T =$	66°	100°	185°	330°
$10^5 \times a/2.3 =$	123	153	996	577

and Table I. summarizes the values of s so computed.

It will be seen that at 66° and 185° the coincidence of observed and computed is satisfactory. At 100° and 330° there are outstanding discrepancies. In the former case (100°) these may be explained away by postulating an initial rod whose hardness is $s = 41$ instead of 39, which would be permissible since in the Balti-

more rods even the hardness $s = 46$ is reached. At 330° such a rod would have to be much too soft; but here small differences of s at the limits of error have a disproportionate effect. On the whole the observations are sufficiently reproduced for the present purposes, since variations in the homogeneity and structure of the rods necessarily detract from the sharpness of the observed values. Moreover, everything depends on the degree to which the *initial* rod may have been under-tempered, as will presently appear, § 5. Finally, it is difficult to define the soft state accurately.

3. *Time Required to Completely Soften Hard Steel at Various Temperatures.*— The observations and the computed curves are given in the following figure.

In the same figure the results of secular or spontaneous softening,

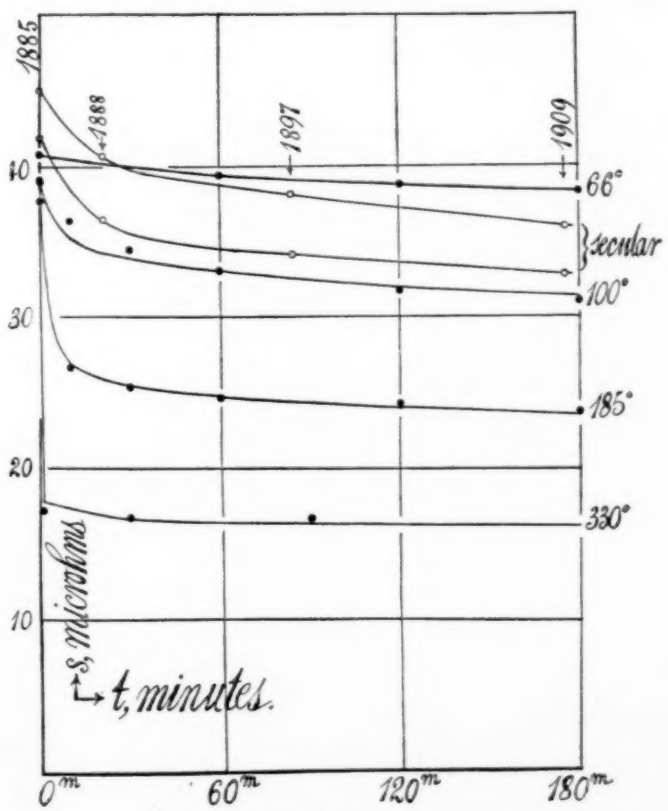

Fig. 1.

for the period of 24 years, may be inscribed, showing how far the decay of hardness has progressed. They will be referred to at greater length in § 4. Clearly, however, the original inference drawn by Doctor Strouhal and myself from observations extending throughout periods of hours, viz., that there is a definite limit of hardness for each temperature, no matter how long the rod may be annealed, is no longer tenable. The limit, when the exposure is prolonged through a period of years, can only be the soft state.

The question thus suggests itself how long it would take the above rods to become as soft as corresponds to temperature at $T = 330°$ where $s = 17.9$. If t_{330} is this time at $T°$

$$t_{330} = \left(\frac{1}{a} \log \frac{1/s_s - 1/s_h}{1/s_s - 1/s} \right)^{T/30}$$

and the results are

$T =$	66°	100°	185°	330°
$t_{330} =$	.45 year	.36 year	.10 year	1 minute

so far as the equation is trustworthy. The times so obtained are smaller than one would anticipate, and without doubt too small. The reason for this is referable to the relatively low value entered for the initial glass-hard state. When observations are conducted through so short a time the degree of glass-hardness from which the tempering proceeds is of marked importance in its bearing on the character of the curves. It will presently appear that the above batch of wires must initially have been below the extreme of hardness.

4. *Temper Varying at Ordinary Temperatures in the Lapse of Time.* — The equation (1) assumed was not a little alluring, inasmuch as it gives at ordinary temperatures $T = 30°$, the simple law of decay $-du/dt = au$. In fact it seemed not unreasonable that with the avoidance of the initial shock, as it were, experienced on plunging hard steel into a high temperature bath the real conditions of decay would appear. But this expectation is necessarily doomed to disappointment, when rods like the present of exceptionally high initial temper are compared with the preceding. By beginning with softer rods, however, the continuity of the distributive equation (1) need not be broken.

Here, if the above test be made by comparing the logarithmetic quotient and t^n where t is now expressed in years, it will be found that n is at least $\frac{1}{2}$. For the thicker rods n is smaller. In both cases the observation after three years is relatively too high, which means probably that the average temperatures of the storage room during these years was higher than during the later years ; but other conditions may have supervened. The observations after 12 and 24 years in the case of the diameters .081 cm. and .044 cm., are practically coincident. I will therefore accept

$$\frac{1}{s} = \frac{1}{s_a} - \left(\frac{1}{s_a} - \frac{1}{s_h}\right) e^{-a\sqrt{t}}$$

for these rods giving $a/2.3 = .0116$ for the former and $a/2.3 = .0115$ for the latter. The thick rods do not conform to the equation. The following table shows the results.

TABLE II.

Spontaneous Softening of Hard Steel. $1/s = 1/s_a - (1/s_a - 1s_h)e^{-a\sqrt{t}}$; t *in Years.*

$t =$	0	3.1	12.2	24.0	
s computed =	44.8	41.1	38.1	36.1	Diameter, .044 cm.
s observed =	44.8	40.6	38.1	36.0	$a/2.3 = .0115$
s computed =	45.0	41.2	38.2	36.0	Diameter, .081 cm.
s observed =	45.0	40.7	38.2	36.1	$a/2.3 = .0116$
s computed =	41.9	37.3	33.9	31.7	Diameter, .126 cm.
s observed =	41.9	36.5	34.2	32.9	$a/2.3 = .0175$

The agreement would have been better if in t^n, n were somewhat smaller than $\frac{1}{2}$. The period which must elapse before these rods soften to the temper corresponding to 330° is $t_{330} = 2{,}840$ years in the first and second cases (diameters .044 and .081) and $t_{330} = 1{,}180$ years in the third (unsatisfactory) case (diameter .126).

For these thick rods the exponent $n = \frac{1}{2}$, as the table shows, is clearly too large. It should even be somewhat smaller than $\frac{1}{3}$. Accepting the latter, the agreement of observed and computed hardness is very close, but the age at which these rods would soften to the temper of 330° naturally becomes much larger in comparison. I omit the data.

5. *The Initial State.* — The behavior of the new rods will repay

special consideration. In case of the thick rods it is possible that the steel was of a different kind (lower in carbon) than the thin batches; but it is more probable that the thick rods do not begin with the glass-hard state, a portion of their substance (the inner layers) being softer. If the part of a section, $l - a$, where a is a fraction, is permanently soft and the part a hard at the beginning,

$$\frac{1}{s} = \frac{1}{s_s} - a\left(\frac{1}{s_s} - \frac{1}{s_h'}\right) e^{-\alpha\sqrt{t}}, \qquad (2)$$

s_h' being the specific resistance of the ideal hard rod. If

$$t = 0, \qquad \frac{1}{s_h} = \frac{(1-a)}{s_s} + \frac{a}{s_h'},$$

and therefore

$$\frac{1}{s} = \frac{1}{s_s} - \left(\frac{1}{s_s} - \frac{1}{s_h}\right) e^{-\alpha\sqrt{t}},$$

reproducing the above equation, which has been found inapplicable for the thick rods.

It is interesting to note, however, that if equation (2) be assumed, it is equivalent to postulating an initial rod of less hardness than the observed value. Thus, for instance, for the thick rods, $a = .964$, whence $\alpha/2.3 = .0121$ and $s_h' = 39.2$ replaces $s_h = 41.9$, with a nearly complete coincidence of observed and computed hardness. It is, moreover, interesting to note that α now has a value agreeing more nearly with the thin rods and the period of softening would be $t_{330} = 2{,}340$ years. The earlier rods of § 2 were thus inadvertently observed along a different part of the complete curve, a part more remote from the origin, and therefore less steep than for the case of the new rods.

In fact, by postulating a softer initial glass-hard rod, about of the same temper as in the cases of §§ 2, 3, it is possible to retain the equation

$$\frac{1}{s} = \frac{1}{s_s} - \left(\frac{1}{s_s} - \frac{1}{s_h}\right) e^{-\alpha t}$$

there inferred, as expressing the case of secular softening at ordinary temperatures. I will omit the data with the remark that the

the time t_{330} of softening would thus also be much reduced, being 343 years for the thin and 293 years for the thick rods, and corresponding to the relatively small values given in § 3 for the initially softer rods discussed. As a matter of fact, however, the present long period of tempering (24 years) shows that an equation with the first power of t is not applicable even when the initial state is disregarded for the logarithmic ratio or criterion continues to bear a curvilinear relation to t.

6. *Conclusion.* — From the results as a whole it follows that the specific resistance s of tempered rods as related to temperature T and time of exposure t may be expressed by an equation of the form

$$\frac{1}{s} = \frac{1}{s_s} - \left(\frac{1}{s_s} - \frac{1}{s_h}\right) e^{-\alpha t^a/T}$$

where a and α are constants and s_s and s_h are the specific resistances of the extremes of soft and glass-hard steel. The value of a depends very essentially on the initial degree of hardness obtained. This equation corresponds to a law of decay with two constants α and a relatively to time t,

$$n = n_0 e^{-\alpha t^a/T},$$

where n is the number of hard molecules after a time t at a temperature T, and where α is a function of temperature. Hence

$$-dn/dt = \frac{a\alpha}{T} n \left(\frac{\alpha}{\log (n_0/n)}\right)^{T/a-1}.$$

Such a law is unfortunately not suggestive. It has thus very little interest beyond the temporary use made of it. It is far removed from the special case $T = a$, or $-dn/dt = an$, so valuable in other similar discussions. Nevertheless it should be accentuated that the sensitiveness of the resistance method must lead to a severe test of the true law of decay whenever it is found. This is elusive in the above cases of spontaneous softening, because the thin rods experience less decay from greater initial hardness, while the temper of the thick rods shows greater decay from less initial hardness. Such an observation is troublesome to reconcile with any usual decay equation.

The conclusion which apparently follows if experiments are conducted for a period of hours, viz., that there is a limiting degree of hardness for each temperature, is not ultimately correct, though it reproduces the phenomenon in its practical aspects. The only final limit is the soft state, and this may be reached at ordinary temperatures, though it takes enormously longer than at higher temperatures. In a general way one may state that if by any miracle the tools in Joseph's carpenter shop, at the beginning of the Christian era, had been glass-hard steel, they would now be so soft as to need retempering to be efficient.

BROWN UNIVERSITY, PROVIDENCE, R. I.

DISPERSION OF ELECTRIC DOUBLE REFRACTION AND ORDINARY DISPERSION IN LIQUIDS.

By Harold E. McComb.

THE discovery and historical development of the work on electric double refraction has been thoroughly discussed by Blackwell.[1] It has been largely confined to a study of carbon bisulphide with, on the whole, very divergent results. For the sake of comparison the following table gives the constant for this liquid as determined by various observers.

TABLE I.

Observer.	Quincke.[2]	Elmén.[3]	Blackwell.	Hagenow.[4]
Wave-length,	590	560	590	590
Constant,	3.27×10^{-7}	4.02×10^{-7}	3.57×10^{-7}	2.85×10^{-7}

(Lemoine[5] gives the value as 3.70×10^{-7} for "red light.")

The wide divergence of these values, which cannot be ascribed to the difference in wave-length used, is difficult to explain. It undoubtedly arises in part from a difference in the carbon bisulphide used but in the writer's opinion it is also to be ascribed in part to inaccurate measurements of some of the experimental factors involved in the determinations. The double refraction in most cases was measured by a Babinet compensator. Elmén and Hagenow used a Brace half-shade system. The potentials were usually furnished by a static machine and measured by an absolute electrometer. Blackwell, however, used a storage battery and should have had little difficulty in measuring the potentials accurately.

The equation used by Elmén for calculating the retardations from the observed rotation of the compensator is incorrect. The obser-

[1] H. L. Blackwell, Proc. Am. Ac. Sci., Vol. 41, p. 647, 1906.
[2] G. Quincke, Wied. Ann., 19, p. 729, 1883.
[3] G. W. Elmén, Phys. Rev., 20, p. 54, 1905.
[4] C. F. Hagenow, Phys. Rev., 27, p. 196, 1908.
[5] J. Lemoine, Compt. Rend., 122, p. 835, 1896.

vations which he made at low potentials are also questionable since they do not agree with the results obtained by Morse,[1] who used the same method later. Koenig[2] has suggested that the apparent departure from Kerr's law, as found by Elmén, might arise from double refraction in the cover glasses. This could hardly be true however, since the retardation is obtained from settings made with the potentials "on" and "off," which would eliminate any effect of this kind.

Blackwell's investigation was primarily to obtain the dispersion, relatively little attention being directed toward obtaining absolute values, while Hagenow sought to establish the absolute value as well as the dispersion.

Havelock[3] has recently developed a general theory for the dispersion of double refraction and shown that it is in agreement with the results of Blackwell and Hagenow. He finds the following relation

$$\frac{B\lambda n_0}{(n_0^2 - 1)^2} = C,$$

in which B is the double refraction constant, λ the wave-length, n_0 is the refractive index, and C is a constant for the substance.

The object of the present investigation was to determine the double refraction constants of various liquids, for various colors, and to compare them with the refractive indices.

Twelve liquids have been investigated, all of which are good insulators, and with the exception of two (chlor-benzol and brom-benzol) none distorted the field of view under the electric strain. There were cases in which the electric force caused the field of view to become blurred, but by successively applying and removing the potential a few times it could always be cleared. All but one of the liquids (ethylene chloride from Eimer and Amend) were obtained from Kahlbaum and only in the case of carbon bisulphide was further purification undertaken. This was usually distilled with $CaCl_2$ and then either filtered through a porous cup into the

[1] L. B. Morse, PHYS. REV., Vol. 23, p. 252, 1906.

[2] W. Koenig, Ann. d. Phys., 28, p. 413, 1909.

[3] T. H. Havelock, Proc. Royal Soc., Vol. 80, p. 28, 1907; PHYS. REV., Vol. 28, p. 136, 1909.

testing cell or distilled directly into the cell without filtration. This difference in treatment did not appreciably change the optical properties.

1. The Optical System.

The optical system was the same as used by Hagenow (l. c.) with a few improvements. The light was more nearly homogeneous, two dispersing prisms being used in place of one and special precautions taken to eliminate extraneous light. Briefly stated, rays from the sun produced a pure spectrum in the plane of the slit S_2 (Fig. 1). By adjusting the prisms P_1P_2 any desired color passed

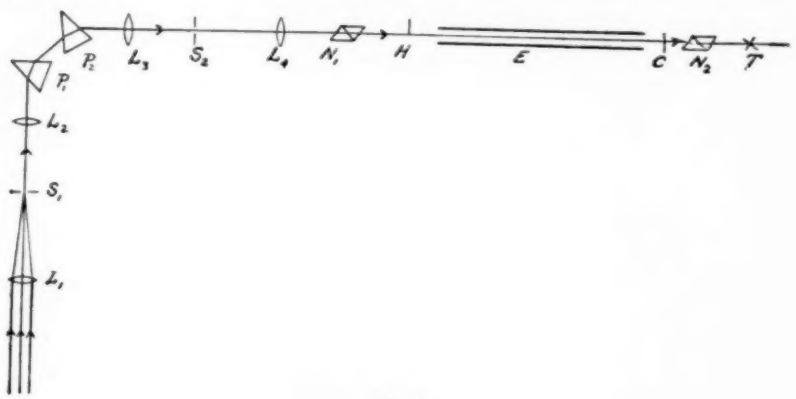

Fig. 1.

through the slit and served as a monochromatic source for making the tests. The polariscope system consisted of a polarizing Nicol N_1 (set at 45° to the lines of force of the electric field in E); a thin mica strip H (with its optic axis at 45° to the plane of polarization) covering half the field; the cell E containing the liquid and the condenser plates; the mica compensator C (covering the whole field) mounted on a graduated circle; the analyzing Nicol N_2 (at right angles to N_1); and the telescope T focused on the half shade H. The electrical connections were made by means of screw contacts which was found to be very essential at low potentials. In some cases the effect of the order of the compensator was eliminated by following Hagenow's method of producing a wave-length retardation; in others the retardation produced by a definite potential difference was measured by the rotation of a compensator.

2. THE ELECTRICAL SYSTEM.

A Holtz machine supplied the potentials ranging from 10,000 to 15,000 volts. The potential difference between the condenser plates was regulated by means of a variable point discharge and measured by a Kaufmann absolute electrometer of the bi-filar type.[1] A mirror was attached to the movable plate and the deflections read by a telescope and scale. The scale was first calibrated by loading the plate with known weights and noting the deflection produced. From this the force of electrical attraction corresponding to an observed deflection was directly obtained and the corresponding potential difference calculated from the formula for an absolute electrometer.[2] The distance between the electrometer plates was adjusted for parallelism and at the same time measured by placing three glass cubes of equal thickness between them and adjusting for contact for all three points at the same time, after which the cubes were removed. This proved more satisfactory than making measurements at different points on the edges of the plates by means of a cathetometer microscope.

For the lower potentials, ranging from 1,400 to 6,000 volts, a series of dynamos was used, each capable of giving a maximum of 1,200 volts. The potential difference was measured by a Weston voltmeter with multipliers giving ranges: 0–150, 0–300, 0–450, 0–600, 0–750, 0–1,500, 0–3,000, 0–4,500, and 0–6,000 volts.

Potentials below 1,400 volts were supplied from a storage battery and also measured by the voltmeter. The storage battery and generators fluctuated so little during an entire set of observations that potentials as high as 6,000 volts were measured to an accuracy of at least .2 per cent.

3. CALIBRATION OF THE COMPENSATOR.

To measure very small retardations it was necessary to calibrate the compensator, for the various wave-lengths used. The method followed was to measure first the potential required to produce a wave-length retardation, then immediately drop down to relatively low potential differences and rotate the compensator for a "match."

[1] Cf. J. E. Almy, Ann. d. Phys. (4), Vol. 1, p. 508, 1900.
[2] Gray's Absolute Measurements, Vol. 1, p. 264.

The potential difference for various angles of the compensator and for various colors was thus obtained. Kerr's law was then applied as follows: Let

$$\delta_1 = B \cdot \frac{V_1^2}{a^2} \cdot l$$

be the retardation produced by a definite low potential difference V_1, l being the length of the condenser plates, a their distance apart, and B the double refraction constant; and

$$\delta_2 = B \cdot \frac{V_2^2}{a^2} \cdot l = 1$$

the retardation produced by the high potential V_2.

Dividing gives

$$\delta_1 = \frac{V_1^2}{V_2^2} \qquad (1)$$

which gives in waves the retardation corresponding to the observed rotation of the compensator.

Representative observations on carbon bisulphide for six different wave-lengths are given in Table II. In the first column are given

TABLE II.

Calibration of Compensator. (CS_2, *Kahlbaum.*) $l = 50$ *cm.* $a = .175$ *cm.*

Wave-length.	One Wave-length Retardation.	Small Retardations.			Order of Compensator. (Degrees.)	Mean Order. (Degrees.)
	Potential Difference. (Electrostatic.)	Potential Difference. (Electrostatic.)	Compensator. (Degrees.)	Retardation. (Waves.)		
440	35.8	4.13	18.7	13.3×10^{-3}	7.50	7.48
		3.63	14.4	10.25	7.46	
460	36.7	2.57	6.94	4.9	7.32	7.34
		4.13	18.0	12.6	7.36	
500	39.75	3.92	15.5	9.71	6.54	6.58
		4.43	19.8	12.4	6.62	
540	41.96	2.61	6.1	3.87	6.16	6.37
		4.47	19.40	11.4	6.58	
600	45.5	2.58	6.05	3.21	5.54	5.58
		3.13	8.75	4.74	5.62	
660	48.5	2.58	5.65	2.82	5.24	5.11
		3.40	10.2	4.91	5.02	
		4.17	15.3	7.39	5.06	

the wave-lengths; in the second the potential differences (as measured by the absolute electrometer) required to produce a wave-length retardation. The third, fourth and fifth columns give respectively the potential differences (as measured by the voltmeter), the corresponding rotations of the compensator for a "match," and the retardations calculated from equation (1).

For this optical system it has been shown by Professor Tuckerman [1] that

$$tg2\pi(N_\lambda + \delta) = -\frac{\sin 2\pi N \sin 2\psi}{1 - 2\sin^2 \pi N \sin^2 2\psi}, \qquad (2)$$

in which N_λ is one half the order of the half-shade mica strip; δ the retardation (in waves) produced by the liquid, N the order of the compensator; and ψ the angle between the principal axis of the compensator and the polarizing Nicol.

Then

$$tg2\pi N_\lambda = -\frac{\sin 2\pi N \sin 2\psi_0}{1 - 2\sin^2 \pi N \sin^2 2\psi_0},$$

in which ψ_0 is the value of ψ for $\delta = 0$. This is best obtained as $\frac{1}{2}$ (the rotation of the compensator between two consecutive "matches" on plane polarized light $-90°$). For the system used $\psi_0 = 6.7°$ for all wave-lengths. Substituting this value of $2\pi N_\lambda$ in equation (2) gives after some reduction

$$tg2\pi\delta = -\sin 2\pi N(\sin 2\psi - \sin 2\psi_0) \times \left[\frac{1 + 2\sin^2 \pi N \sin^2 2\psi_0 \sin 2\psi}{(1 - 2\sin^2 \pi N \sin^2 2\psi)(1 - 2\sin^2 \pi N \sin^2 2\psi_0) + \sin^2 2\pi N \sin 2\psi \sin 2\psi_0}\right].$$

For the compensator here used $\sin 2\pi N$ is small. Expanding to quantities of the second order in $\sin 2\pi N$ gives

$$\delta = -N(\sin 2\psi - \sin 2\psi_0)\{1 - \tfrac{1}{6}\sin^2 2\pi N[1 - (\sin 2\psi - \sin 2\psi_0)^2] + \text{etc.}\}.$$

Neglecting the quantity in the brackets introduces an error not greater than .3 per cent. for the compensator used, hence the equation in the following form may be used

$$\delta = N[\sin 2(\varphi - \varphi_0) + \sin 2\varphi_0], \qquad (3)$$

[1] L. B. Tuckerman, Jr., Univ. of Nebr. Studies, April, 1909.

in which $\varphi = -(\phi - \phi_0)$ is the rotation of the compensator between "matches" obtained respectively with potentials "on" and "off," and $\varphi_0 = \phi_0$.

Using this formula the value of N was calculated for the different wave-lengths from the results of the calibration given in column 5 of Table II. In column 6 are recorded these values of N and column 7 gives the means. In Fig. 2 is plotted the curve repre-

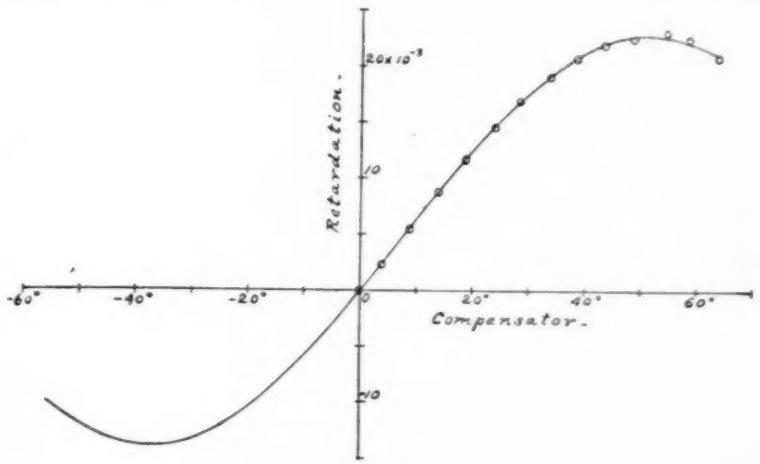

Fig. 2.

sented by equation (3) while the points located on it represent the corresponding values obtained by calibration. The equation is thus experimentally verified.

In addition to the values obtained with carbon bisulphide, which required a high potential difference to produce a wave-length retardation, observations were also made on nitro benzol in which the effect is about 100 times as large for the same potential difference. On account of the absorption in the violet a shorter cell was required so that a potential difference of about 2,400 volts gave a wave-length retardation. The potential was supplied by two of the dynamos in series and measured by the voltmeter. Observations made on a wave-length 500 $\mu\mu$ gave for the order of the compensator the value 6°.8 as against 6.6 in the other test. Each liquid has for the calibration its weak point as well as its strong. Carbon bisulphide is quite transparent and the compensator settings were very accurate

even in the violet, but the high potentials could not be measured to an accuracy less than one per cent. With nitro benzol the potentials (measured by the voltmeter) were accurate in all cases to .2 per cent., but the depolarization in this liquid at potentials sufficient to produce a wave-length retardation reduced considerably the accuracy of the compensator settings. The two calibrations for the color tested agree fairly well but the results from the carbon bisulphide being regarded the better were used. The dispersion in the mica as given in column 7 of Table II. and plotted in Fig. 3 agrees

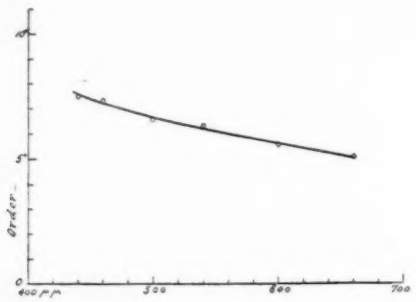

Fig. 3.

very closely with that obtained in this laboratory by Mr. Tool who used an entirely different method.

4. END EFFECTS.

A correction for the effect at the ends of the condenser was necessary, especially when short ones were used. To determine this correction measurements were made on the same liquid with three different lengths (2, 15 and 50 cm.). It was found that the same small correction applied to each length gave consistent values for the double refraction constant. This correction was found to be approximately .18 cm.; it is therefore important when short condensers are used. The following table (III.) shows the value of this correction as obtained from different tests.

5. REFRACTIVE INDICES AND ABSORPTION.

The refractive indices were determined by means of the spectrometer and are accurate to .0002. Observations were made on

TABLE III.

End Effects.

Liquid.	Length of Condenser. (cm.)	End Correction. (cm.)
Di-methyl aniline.	15	
	2	.19
Nitro toluol.	50	
	15	.17
Nitro toluol.	50	
	2	.19
	Mean	.18

exactly the same colors used in the double refraction measurements at practically the same temperature and usually directly afterwards.

The absorption region of the various liquids was also incidentally obtained. This was done by first photographing the spectrum of an iron arc produced by a Rowland concave diffraction grating. A photograph under the same conditions was then taken with the liquid (.4 cm. thick — in a quartz cell) placed between the arc and the slit of the spectroscope. The photographs through air gave lines ranging from 600 to 225 $\mu\mu$. All exposures were for the same length of time. An absorption region when present was found in all the liquids to be very abrupt.

6. EXPERIMENTAL RESULTS.

For the purpose of testing the reliability and accuracy of the system as a whole repeated observations were made on carbon bisulphide. In Table IV. are given the values of the constant B for commercial carbon bisulphide as obtained with potentials sufficient to produce a wave-length retardation. The four different columns represent tests with four different samples from the same bottle. The sixth and seventh columns give respectively the mean values of B as obtained from these results, and the corresponding values obtained from Hagenow's curve.[1] The close agreement with his furnishes a satisfactory check on the method. The last two columns give respectively the refractive indices and the value of B as calculated from Havelock's equation (page 544). The value of C given at the head of the column was the mean obtained by applying that equation to the experimental values in column 6, in which determination the unreliable values were omitted.

[1] Though not stated in his paper, Mr. Hagenow informed me that he used commercial CS_2.

Table V. gives the values for Kahlbaum's carbon bisulphide under the same treatment as the commercial. The values of B are throughout the spectrum somewhat greater for Kahlbaum's than for the

TABLE IV.

Carbon Bisulphide (CS_2). (*Commerical, distilled.*) $l = 50$ *cm.* $a = .175$ *cm.*

Wave-length. ($\mu\mu$)	B. (22° C.)				Mean. B	B (Hagenow.)	Refractive Index. (23°.5 C.)	B Calculated. ($C=1.024\times10^{-11}$.)
	1.	2.	3.	4.				
440	4.50×10^{-7}	4.56×10^{-7}	4.17×10^{-7}	4.36×10^{-7}	4.40×10^{-7}	4.35×10^{-7}	1.6672	4.42×10^{-7}
460	4.14	4.35	4.12	4.05	4.16	4.05	1.6586	4.12
480	3.88	4.09	3.91	3.83	3.92	3.78	1.6504	3.83
500	3.68	3.80	3.62	3.56	3.65	3.58	1.6439	3.61
520	3.45	3.54	3.43	3.67	3.52	3.39	1.6380	3.41
540	3.26	3.33	3.23	3.17	3.25	3.19	1.6330	3.23
560	3.10	3.17	3.07	3.03	3.09	3.04	1.6286	3.07
580	2.95	3.08	2.90	2.89	2.95	2.90	1.6250	2.93
600	2.81	2.83	2.79	2.78	2.80	2.76	1.6218	2.80
630	2.65	2.65	2.63	2.60	2.63	2.60	1.6172	2.62
660	2.51	2.51	2.47	2.47	2.49	2.47	1.6138	2.46

TABLE V.

Carbon Bisulphide (CS_2). (*Kahlbaum, distilled.*) $l = 50$ *cm.* $a = .175$ *cm.*

Wave-length. ($\mu\mu$)	B. (22°.5 C.)			Refractive Index. (23°.5 C.)	B Calculated. ($C=1.114\times10^{-11}$.)
	1.	2.	Mean.		
440	4.87×10^{-7}	4.78×10^{-7}	4.82×10^{-7}	1.6672	4.81×10^{-7}
460	4.52	4.55	4.53	1.6586	4.48
480	4.28		4.28	1.6504	4.18
500	3.95	3.89	3.92	1.6439	3.93
520	3.69		3.69	1.6380	3.71
540	3.52	3.49	3.51	1.6330	3.51
560	3.36		3.36	1.6285	3.34
580	3.17		3.17	1.6250	3.19
600	3.03	2.96	3.00	1.6218	3.05
630	2.88		2.88	1.6172	2.85
660	2.66	2.61	2.64	1.6138	2.69

commercial while the refractive indices are practically the same. Havelock's equation fits the two equally well.

Tables VI. to XIII. give similar results for various liquids belonging to the benzene series in which as in all that follow the retardations were measured by rotating the compensator and applying

TABLE VI.

Benzol (C_6H_6). $l = 50$ *cm.* $a = .175$ *cm.*

Wave-length. ($\mu\mu$)	Potential Difference. (Electro-static.)	Compen-sator. (Degrees.)	Retardation. (Waves.)	B (23°.7 C.)	Refrac-tive Index. (24° C.)	B Calculated. ($C = .329 \times 10^{-11}$.)
440	10.05	19.65	14.0×10^{-3}	$.845 \times 10^{-7}$	1.5191	$.842 \times 10^{-7}$
460	10.05	19.35	13.0	.780	1.5150	.792
480	10.10	19.40	12.7	.760	1.5113	.748
500	10.12	19.18	12.0	.710	1.5081	.708
520	10.05	18.22	11.1	.660	1.5054	.673
540	10.05	18.10	10.7	.640	1.5031	.644
560	10.05	17.60	10.1	.600	1.5009	.615
580	10.00	17.80	9.8	.595	1.4992	.582
600	10.00	18.10	9.7	.590	1.4974	.564
630	10.05	17.60	9.0	.540	1.4953	.533
660	10.05	17.40	8.5	.510	1.4935	.505

TABLE VII.

Nitro Benzol ($C_6H_5 \cdot NO_2$). $l = 2.18$ *cm.* $a = .171$ *cm.*

Wave-length. ($\mu\mu$)	Potential Difference. (Electro-static.)	Compen-sator. (Degrees.)	Retardation. (Waves.)	B (22°.5 C.)	Refractive Index. (25° C.)	B Calculated. ($C = 117 \times 10^{-11}$.)
440	2.07	17.23	12.4×10^{-3}	388×10^{-7}	1.5829	381×10^{-7}
460	2.07	16.70	11.4	356	1.5759	355
480	2.07	16.12	10.7	333	1.5696	333
500	2.07	15.77	9.95	311	1.5645	314
520	2.07	15.60	9.60	296	1.5602	296
540	2.08	14.95	8.90	275	1.5566	283
560	2.08	15.00	8.65	269	1.5533	274
580	2.07	14.73	8.20	256	1.5506	255
600	2.07	14.47	7.80	242	1.5481	246
630	2.07	14.24	7.35	230	1.5449	231
660	2.07	14.10	6.90	216	1.5424	218

equation (3). It was difficult to obtain accurate results on chlor- and brom-benzol on account of motions in the liquid produced by the electric strain. The constant for nitro benzol decreases gradually under electric strain so that it was found necessary to make corrections for this change during a series of observations.

TABLE VIII.

Nitro Toluol ($C_6H_5 \cdot CH_2 \cdot NO_2$). $l = 2.18$ *cm.* $a = .171$ *cm.*

Wave-length. ($\mu\mu$)	Potential Difference. (Electro-static.)	Compen-sator. (Degrees.)	Retardation. (Waves.)	B (24° C.)	Refractive Index. (22° C.)	B Calculated. ($C = 53.7 \times 10^{-11}$
440	3.10	(16.00)	11.6×10^{-3}	(161×10^{-7})	1.5757	171×10^{-7}
460	3.10	16.65	11.4	158	1.5695	160
480	3.09	15.78	10.5	147	1.5635	150
500	3.10	15.88	10.0	139	1.5589	141
520	3.11	15.92	9.75	134	1.5549	134
540	3.11	15.62	9.25	127	1.5516	127
560	3.12	15.46	8.9	122	1.5485	121
580	3.12	15.84	8.8	121	1.5463	116
600	3.13	15.31	8.25	112	1.5439	111
630	3.13	15.31	7.85	107	1.5409	104
660	3.13	14.76	7.25	99	1.5386	100

Observations in parenthesis are of questionable accuracy.

TABLE IX.

Di-methyl Aniline [$C_6H_5N(CH_3)_2$]. $l = 2.18$ *cm.* $a = .171$ *cm.*

Wave-length. ($\mu\mu$)	Potential Difference. (Electro-static.)	Compen-sator. (Degrees.)	Retardation. (Waves.)	B (25° C.)	Refractive Index. (26° C.)	B Calculated. ($C = 4.34 \times 10^{-11}$.)
440	10.73	18.28	13.1×10^{-3}	15.4×10^{-7}	1.5875	14.4×10^{-7}
460	10.57	17.07	11.7	13.4	1.5808	13.4
480	10.53	16.10	10.7	12.8	1.5747	12.6
500	10.60	15.63	9.85	11.8	1.5696	11.9
520	10.58	15.95	9.80	11.7	1.5654	11.2
540	10.67	15.35	8.95	10.5	1.5620	10.7
560	10.67	15.15	8.75	10.3	1.5588	10.2
580	10.60	14.92	8.3	9.8	1.5562	9.7
600	10.60	14.43	7.8	9.3	1.5538	9.3
630	10.64	14.21	7.3	8.6	1.5506	8.8
660	10.60	13.81	6.8	8.1	1.5482	8.3

TABLE X.

Di-ethyl Aniline [$C_6H_5N(C_2H_5)_2$]. $l = 2.18$ *cm.* $a = .171$ *cm.*

Wave-length. ($\mu\mu$)	Potential Difference. (Electro-static.)	Compen-sator. (Degrees.)	Retardation. (Waves.)	B (25°.2 C.)	Refractive Index. (26° C.)	B Calculated. ($C = 4.84 \times 10^{-11}$.)
440	10.25	16.60	12.0×10^{-3}	15.3×10^{-7}	1.5668	14.9×10^{-7}
460	10.23	15.81	10.8	13.8	1.5608	13.9
480	10.22	15.52	10.3	13.2	1.5555	13.1
500	10.22	(14.23)	(9.0)	(11.5)	1.5510	12.3
520	10.18	15.13	9.3	12.0	1.5473	11.7
540	10.20	14.70	8.7	11.2	1.5443	11.1
560	10.17	13.85	8.0	10.4	1.5414	10.6
580	10.13	13.50	7.5	9.8	1.5390	10.1
600	10.18	13.72	7.4	9.5	1.5369	9.75
630	10.15	13.52	7.0	9.1	1.5341	9.2
660	10.20	13.60	6.7	8.6	1.5319	8.7

Observations in parenthesis are of questionable accuracy.

TABLE XI.

α-mono-brom-naphtalene ($C_{10}H_7Br$). $l = 2.18$ *cm.* $a = .171$ *cm.*

Wave-length. ($\mu\mu$)	Potential Difference. (Electrostatic.)	Compensator. (Degrees.)	Retardation. (Waves.)	B (21°.5 C.)	Refractive Index. (26°.3 C.)	B Calculated. ($C = 2.83 \times 10^{-11}$.)
440	11.40	18.18	12.8×10^{-3}	13.2×10^{-7}	1.6970	13.4×10^{-7}
460	11.40	17.60	12.0	12.4	1.6885	12.5
480	11.40	16.90	11.2	11.5	1.6802	11.6
500	11.40	17.05	10.7	11.0	1.6738	11.0
520	11.45	16.50	9.85	10.2	1.6683	10.6
540	11.30	15.80	9.4	9.85	1.6638	9.87
560	11.30	15.70	9.1	9.5	1.6596	9.35
580	11.30	15.35	8.5	9.0	1.6560	8.9
600	11.30	15.43	8.3	8.7	1.6529	8.6
630	11.25	14.65	7.5	7.9	1.6487	8.05
660	11.25	(15.08)	(6.9)	(7.3)	1.6454	7.6

Observations in parenthesis are of questionable accuracy.

TABLE XII.

Chlor-Benzol (C_6H_5Cl). $l = 15.2$ *cm.* $a = .175$ *cm.*

Wave-length. ($\mu\mu$)	Potential Difference. (Electro-static.)	Compen-sator. (Degrees.)	Retardation. (Waves.)	B (24° C.)	Refrac-tive Index. (25°.5 C.)	B Calculated. ($C = 4.74 \times 10^{-11}$.)
440	4.08	15.26	11.0×10^{-3}	13.3×10^{-7}	1.5428	13.2×10^{-7}
460	4.08	15.10	10.4	12.6	1.5386	12.5
480	4.08	14.45	9.6	11.6	1.5348	11.8
500	4.08	14.69	9.25	11.2	1.5314	11.2
520	4.08	13.78	8.45	10.2	1.5286	10.7
540	4.08	13.75	8.20	9.9	1.5264	10.2
560	4.10	14.75	8.50	10.2	1.5241	9.7
580	4.10	14.63	8.10	9.8	1.5222	9.3
600	4.10	12.75	6.90	8.3	1.5207	8.9
630	4.10	13.62	7.0	8.6	1.5185	8.5
660	4.10	14.05	6.9	8.3	1.5164	8.0

TABLE XIII.

Brom-Benzol (C_6H_5Br). $l = 15.2$ *cm.* $a = .175$ *cm.*

Wave-length. ($\mu\mu$)	Potential Difference. (Electro-static.)	Compen-sator. (Degrees.)	Retardation. (Waves.)	B (24°.9 C.)	Refrac-tive Index. (25° C.)	B Calculated. ($C = 4.12 \times 10^{-11}$.)
440	4.27	18.40	13.2×10^{-3}	14.6×10^{-7}	1.5797	13.3×10^{-7}
460	4.30	18.06	12.3	13.4	1.5751	12.5
480	4.27	(16.00)	(9.8)	(10.9)	1.5708	11.8
500	4.27	16.10	10.2	11.3	1.5671	11.2
520	4.27	15.20	9.35	10.4	1.5639	10.6
540	4.27	15.20	9.00	10.0	1.5611	10.1
560	4.27	15.20	8.8	9.9	1.5585	9.65
580	4.27	(13.10)	(7.3)	(8.1)	1.5565	9.25
600	4.20	14.40	7.7	8.8	1.5547	8.9
630	4.27	(16.10)	(8.3)	(9.2)	1.5522	8.4
660	4.27	14.90	7.3	8.1	1.5501	7.9

Observations in parenthesis are of questionable accuracy.

Tables XIV., XV. and XVI. give the constants for liquids belonging to another series and of lower dispersion.

Table XVII. gives for comparison the dispersion of double refraction and the wave-length as shown by the photographs at which abrupt absorption begins. The dispersion was obtained by dividing the difference between the values of the constant B at 440 and 660 $\mu\mu$ by its value at 590. No relation between the

TABLE XIV.

Chloroform ($CHCl_3$). $l = 15.2$ *cm.* $a = .175$ *cm.*

Wave-length. ($\mu\mu$)	Potential Difference (Electro-static.)	Compen-sator. (Degrees.)	Retardation. (Waves.)	B (23° C.)	Refractive Index. (24°.5 C.)	B Calculated. ($C = -2.40 \times 10^{-11}$.)
440	6.60	−14.70	-9.30×10^{-3}	-4.32×10^{-7}	1.4536	-4.66×10^{-7}
460	6.60	−14.85	−9.05	−4.20	1.4514	−4.41
480	6.60	−14.90	−8.65	−4.01	1.4495	−4.19
500	6.61	−14.77	−8.20	−3.78	1.4479	−4.06
520	6.61	−15.90	−8.45	−3.90	1.4464	−3.80
540	6.61	−15.10	−7.82	−3.72	1.4452	−3.63
560	6.62	−15.20	−7.65	−3.53	1.4440	−3.48
580	6.65	−15.40	−7.47	−3.41	1.4430	−3.36
600	6.66	−14.90	−7.02	−3.21	1.4421	−3.24
630	6.60	−14.60	−6.60	−3.05	1.4412	−3.07
660	6.61	−14.65	−6.30	−2.91	1.4400	−2.91

TABLE XV.

Amyl Chloride [$CH_3(CH_2)_2CH_2Cl$]. $l = 15.2$ *cm.* $a = .175$ *cm.*

Wave-length. ($\mu\mu$)	Potential Difference. (Electro-static.)	Compen-sator. (Degrees.)	Retardation. (Waves.)	B (23° C.)	Refrac-tive Index. (24°.2 C.)	B Calculated. ($C = 2.77 \times 10^{-11}$.)
440	7.07	15.56	11.2×10^{-3}	4.52×10^{-7}	1.4155	4.49×10^{-7}.
460	7.07	15.18	10.4	4.21	1.4138	4.26
480	7.07	14.80	9.8	3.96	1.4123	4.03
500	7.07	15.01	9.5	3.85	1.4109	3.86
520	7.07	14.85	9.2	3.73	1.4097	3.69
540	7.07	14.83	8.8	3.56	1.4088	3.53
560	7.07	14.87	8.6	3.47	1.4077	3.39
580	7.07	14.48	8.1	3.27	1.4068	3.26
600	7.09	14.45	7.8	3.15	1.4062	3.14
630	7.09	14.15	7.3	2.94	1.4052	2.97
660	7.08	13.93	6.85	2.76	1.4044	2.83

magnitude of the dispersion and the position of the absorption region is apparent.

The theoretical equation given by Havelock is within experimental errors in agreement with the results obtained for the various liquids. This being true it is to be preferred to the empirical relation published in an abstract of this work a few months ago,[1] which stated: for a given liquid the slope of the B-curve is proportional

[1] H. E. McComb, PHYS. REV., Vol. 27, p. 336, 1908.

TABLE XVI.

Ethylene Chloride ($C_2H_4Cl_2$). $l = 15.2$ *cm.* $a = .175$ *cm.*

Wave-length. ($\mu\mu$)	Potential Difference. (Electro-static.)	Compen-sator. (Degrees.)	Retardation. (Waves.)	B (18°.6 C.)	Refractive Index. (19°.4 C.)	B Calculated. ($C = 3.26 \times 10^{-11}$.)
440	6.40	17.98	12.9×10^{-3}	6.34×10^{-7}	1.4551	6.35×10^{-7}
460	6.37	17.09	11.7	5.82	1.4532	6.03
480	6.37	17.32	11.4	5.68	1.4512	5.72
500	6.40	17.47	10.9	5.42	1.4498	5.47
520	6.40	17.33	10.6	5.22	1.4484	5.21
540	6.41	17.30	10.2	5.02	1.4473	5.00
560	6.43	17.25	9.9	4.86	1.4461	4.79
580	6.45	17.52	9.75	4.78	1.4452	4.61
600	6.43	17.82	9.5	4.63	1.4444	4.43
630	6.43	17.01	8.7	4.30	1.4432	4.21
660	6.43	17.08	8.3	4.06	1.4424	4.00

TABLE XVII.

Absorption Region and Dispersion.

Liquid.	Dispersion of Electric Double Refraction.	Wave-length at which Total Absorption Begins.
Carbon bisulphide,	.682	372 $\mu\mu$
α-mono-brom naphtalene,	.662	345
Nitro benzol,	.652	420
Di-methyl aniline,	.641	346
Di-ethyl aniline,	.625	346
Nitro toluol,	.622	425
Brom benzol,	.593	340
Benzol,	.588	275
Chlor benzol,	.571	none.
Chloroform,	.530	none.
Ethylene chloride,	.520	340
Amyl chloride,	.518	none.

throughout the spectrum to the slope of the refractive index curve at the same wave-length.

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DISPERSION OF MAGNETIC DOUBLE REFRACTION IN LIQUIDS COMPARED WITH THAT OF ELECTRIC DOUBLE REFRACTION

By C. A. Skinner.

THAT a number of liquids when placed in a magnetic field are plane doubly refracting to light propagated in a direction normal to the field and that this property follows Kerr's law for corresponding double refraction in an electric field has been discovered by Cotton and Mouton.[1] These physicists, investigating nitro benzol, also found that within experimental errors the dispersion of double refraction is the same in the magnetic as in the electric field.

In this paper are given the results of a simultaneous investigation of the double refraction of those liquids in a magnetic field which Mr. McComb has been studying in an electric.[2] In addition to the plane double refraction normal to the field the circular double refraction (rotation) along the lines of force was also incidentally measured. Differential absorption (dichroism) both along and across the lines of force was carefully sought for but nothing found.

The results corroborate in general the conclusions of Cotton and Mouton, and, as shown by McComb for electric double refraction, the form of the dispersion curve fits in most cases — within experimental errors — the theory of Havelock.[3]

Experimental Arrangement.

The apparatus was about the same as that recently used for measuring magnetic rotation and ellipticity produced by metal films.[4] For measuring rotation the arrangement was exactly the same. Light from the sun as source passed successively through a

[1] A. Cotton and H. Mouton, Compt. Rend., July and Nov., 1907, and July, 1908.

[2] See preceding paper.

[3] T. H. Havelock, Proc. Roy. Soc., Vol. 80, p. 28, 1907. Phys. Rev., Vol. 28, p. 136, 1909.

[4] C. A. Skinner and A. Q. Tool, Phil. Mag., December, 1908.

Fuess spectral instrument (for giving the desired wave-length), a collimating lens, a Nicol prism, the liquid studied — placed between the pierced pole-pieces of the electromagnet — and a Lippich analyzing system mounted on a graduated circle.

For measuring the relative retardation normal to the field the electromagnet was placed in the proper position and provided with new pole pieces possessing faces 6 cm. long and 1 cm. wide — with an air gap of about 7 mm. The Nicol was set to give a vibration of the transmitted light at 45° to the lines of force and the Lippich system replaced by a Brace elliptic analyzing system consisting of a very thin strip of mica (with its optic axis at 45° to the plane of polarization) covering half the field of view, followed by a somewhat thicker sheet of mica (the compensator) covering the whole field and carried by a graduated circle. The relative retardation of the vibration perpendicular to the lines of force over that parallel to them was obtained from the formula [1]

$$\delta = N\,[\sin 2(\varphi - \varphi_0) + \sin 2\varphi_0], \qquad (1)$$

in which δ = retardation in waves; N = the order of the compensator (.0181 wave for 560 $\mu\mu$); φ = the angle of rotation of compensator from position of "match" on plane polarized light to that on the light analyzed; φ_0 = the angle of rotation of the compensator from a "match" on the plane polarized light to the nearest position in which its optic axis is parallel (or perpendicular) to the plane of polarization. The magnitude of this angle φ_0 in the instrument used was 4.9°. Opposite rotations for φ and φ_0 give opposite signs to their magnitudes. The sign of φ serves to determine (from a previous test of the compensator) which of the two components of the light vibration is propagated with the slower velocity.

For tests normal to the field the liquid was placed in a cell made of glass tubing of rectangular cross-section (6 × 11 mm.). The length of the light path in the liquid was 6 cm. in all cases where the absorption was not too great.

The strength of the field in the space between the center of the pole faces was calculated for various magnetizing currents from the

[1] The general theory of this instrument has recently been published by Professor Tuckerman, University of Nebraska Studies, April, 1909. The deduction of this formula from his general equation is given by McComb (*l. c.*).

rotation (of light passing along the lines of force) produced by a given length of carbon bisulphide — the poles having been pierced at that point for the purpose. The mean field strength over the 6 cm. length of pole face was obtained therefrom by comparing the double refraction (normal to the field) of a 6 cm. cell with that of a 1.5 cm. cell placed at the center of the pole faces. One of the liquids producing the largest effect (α-mono-brom-naphthalene) was used for this purpose. It was found that the mean field strength sought was .97 that at the center of the pole faces.

Experimental Results.

In the following tables (I., II., III.) are given the observed relation between relative retardation and field strength (c.g.s.) for three different liquids showing the largest effects.

Table I.

Nitro Benzol. Wave-length, 500 $\mu\mu$.

Field (H)	9,050	10,100	10,900	11,750	13,750
Retardation (δ)	1.69×10^{-3}	2.11×10^{-3}	2.46×10^{-3}	2.93×10^{-3}	3.89×10^{-3}
δ/H^2	2.07×10^{-13}	2.07×10^{-13}	2.07×10^{-13}	2.11×10^{-13}	2.06×10^{-13}

Table II.

Nitro Toluol. Wave-length, 500 $\mu\mu$.

Field (H)	9,000	9,950	10,900	11,750	13,750
Retardation (δ)	1.15×10^{-3}	1.38×10^{-3}	1.63×10^{-3}	1.96×10^{-3}	2.57×10^{-3}
δ/H^2	1.42×10^{-13}	1.39×10^{-13}	1.39×10^{-13}	1.42×10^{-13}	1.36×10^{-13}

Table III.

α-mono-brom-naphthalene. Wave-length, 500 $\mu\mu$.

Field (H)	8,900	10,050	10,850	11,750	13,800
Retardation (δ)	1.71×10^{-3}	2.19×10^{-3}	2.60×10^{-3}	3.04×10^{-1}	4.15×10^{-1}
δ/H^2	2.17×10^{-13}	2.17×10^{-13}	2.21×100^{-13}	2.20×10^{-13}	2.18×10^{-13}

These results show that the fluctuation of the ratio of retardation to the square of the field strength is such that it is to be explained as arising from experimental errors, hence they corroborate the statement made by Cotton and Mouton that Kerr's law for electric double refraction is also applicable to the magnetic double refraction of these liquids.

With this law as a basis the double refraction constants of the different liquids at various wave-lengths were obtained. This law states that

$$\delta = B_m H^2 l, \tag{2}$$

δ being the relative retardation in waves produced by a length l of the liquid in a field of intensity H (c.g.s.). B_m is then defined as the retardation produced by unit length of liquid in a unit field, or

$$B_m = \left(\frac{1}{\lambda_S} - \frac{1}{\lambda_P}\right)_{H=1} \tag{3}$$

defines it so that a positive value of B_m indicates that the component vibration parallel to the direction of the field is propagated with a velocity greater than that of the perpendicular component, λ_S and λ_P representing the wave-lengths in the liquid of the perpendicular and parallel components respectively.

TABLE IV.

Benzol.

Wave-length. ($\mu\mu$)	Along Field (12,500).	Across Field (13,800 c.g.s.). Temperature 26.5° C.					
	Rotation. $\left(\frac{\text{degrees}}{\text{cm.}\times H}\right)$	Compensator. (Degrees.)	Retardation. (Waves.)	B_m.	B_e (McComb.)	B_m/B_e	B_m Calculated. ($C=4.12\times10^{-10}$.)
440	1.00×10^{-3}	1.52	1.14×10^{-3}	10.0×10^{-13}	$.84\times10^{-7}$	11.9×10^{-6}	10.4×10^{-13}
460	.89	1.41	1.05	9.2	.78	11.8	9.9
480	.83	1.50	1.08	9.5	.76	12.4	9.5
500	.75	1.49	1.02	8.9	.71	12.5	8.8
520	.70	1.45	.95	8.3	.66	12.5	8.3
540	.63	1.39	.89	7.8	.64	12.2	8.0
560	.57	1.35	.86	7.5	.60	12.5	7.6
580	.53	1.43	.86	7.5	.59	12.7	7.4
600	.49	1.37	.83	7.3	.59	12.4	7.1
630	.44	1.41	.79	6.9	.54	12.8	6.6
660	.39	1.41	.76	6.7	.51	13.1	6.3

In Tables IV. to XVII. are incorporated the results of the investigation of dispersion, as indicated by the values of B_m, throughout the visible spectrum. From the observed rotation of the compen-

TABLE V.

Nitro Benzol.

Wavelength. ($\mu\mu$)	Along Field (13,200). Rotation. $\left(\frac{\text{degrees}}{\text{cm.}\times H}\right)$	Across Field (13,600 c.g.s.). Temperature 24° C. Compensator. (Degrees.)	Retardation. (Waves.)	B_m	B_e (McComb.)	B_m/B_e	B_m Calculated. ($C=13.1\times10^{-10}$.)
440	$.66\times10^{-3}$	5.90	4.49×10^{-3}	40.5×10^{-13}	388×10^{-7}	$.104\times10^{-6}$	42.1×10^{-13}
460	.60	5.82	4.30	38.7	356	.108	40.0
480	.56	5.82	4.15	37.5	333	.112	37.2
500	.51	5.64	3.86	34.9	311	.111	35.2
520	.48	5.58	3.71	33.5	296	.113	33.5
540	.445	5.52	3.56	32.1	275	.116	31.5
560	.415	5.41	3.41	30.8	269	.114	30.2
580	.38	5.29	3.22	29.0	256	.113	28.6
600	.36	5.21	3.06	27.6	242	.114	27.4
630	.325	5.14	2.91	26.3	230	.114	26.1
660	.295	5.10	2.74	24.6	216	.114	24.4

TABLE VI.

Nitro Toluol.

Wavelength. ($\mu\mu$)	Along Field (12,100). Rotation. $\left(\frac{\text{degrees}}{\text{cm.}\times H}\right)$	Across Field (13,800 c.g.s.). Temperature 25° C. Compensator. (Degrees.)	Retardation. (Waves.)	B_m	B_e (McComb.)	B_m/B_e	B_m Calculated. ($C=7.96\times10^{-10}$.)
440	$.65\times10^{-3}$	3.70	2.83×10^{-3}	24.9×10^{-13}	161×10^{-7}	$.154\times10^{-6}$	25.1×10^{-13}
460	.61	3.56	2.65	23.3	158	.147	23.5
480	.57	3.46	2.50	22.0	147	.150	22.3
500	53	3.40	2.37	20.8	139	.150	21.0
520	.475	3.37	2.24	19.6	134	.147	19.6
540	.445	3.29	2.13	18.7	127	.147	18.8
560	.41	3.23	2.05	18.0	122	.147	17.9
580	.375	3.28	2.01	17.6	121	.145	17.3
600	.35	3.20	1.89	16.6	112	.148	16.5
630	.315	3.12	1.77	15.5	107	.145	15.6
660	.285	3.16	1.71	15.0	99	.151	14.6

sator the retardation δ for the wave-length considered was calculated by equation (1), and from this the value of B_m was obtained by the use of (2). The ratio of B_m to the corresponding constant of electric double refraction (B_e) as obtained by McComb is also given in those cases where the liquid furnished an appreciable or reliable value for the latter constant. Finally, from the refractive indices as determined by McComb (l. c.) the value of B_m was calculated from Havelock's formula

$$\frac{B_m \lambda n_0}{(n_0^2 - 1)^2} = C, \qquad (4)$$

n_0 being the refractive index, λ the wave-length (*in vacuo*), and C a constant for the liquid. The value of C used in the calculation in each case is given at the top of the column.

TABLE VII.

Aniline.

Wave-length. ($\mu\mu$)	Along Field (14,200).	Across Field (13,800 c.g.s.). Temperature 25° C.				
	Rotation. $\left(\frac{\text{degrees}}{\text{cm.}\times H}\right)$	Compensator. (Degrees.)	Retardation. (Waves.)	B_m	Refractive Index.	B_m Calculated. ($C=2.04\times10^{-10}$.)
440	1.49×10^{-3}					
460	1.31	1.04	$.77\times10^{-3}$	6.7×10^{-13}	1.6114	7.1×10^{-13}
480	1.18	.97	.69	6.1	1.6056	6.5
500	1.05	1.04	.71	6.2	1.6008	6.1
520	.95	1.00	.65	5.7	1.5967	5.9
540	.86	.97	.63	5.5	1.5931	5.6
560	.78	.97	.60	5.3	1.5898	5.3
580	.71	.97	.58	5.1	1.5872	5.1
600	.65	1.03	.60	5.3	1.5847	4.9
630	.59	.98	.55	4.8	1.5815	4.6
660	.54	.99	.53	4.6	1.5789	4.4

Chloroform and bromoform give appreciable magnitudes for the electric double refraction — both negative. In the magnetic field they gave also an appreciable negative value for the double refraction constant, but too small to measure with any satisfactory accuracy. Amyl chloride which is positive in the electric field did not show an appreciable effect in the magnetic field. Ethylene chloride which, according to McComb, gives reliable values in the electric field was found to possess in the magnetic field a value

which diminished so rapidly with time that no satisfactory measurements were made.

Aniline (Table VII.) mono-ethyl aniline (Table IX.) and toluidin (Table XVII.) gave measurable results in the magnetic field but no observable effect in the electric.

TABLE VIII.

Di-methyl Aniline.

Wave-length. ($\mu\mu$)	Along Field (12,900).	Across Field (13,800 c.g.s.). Temperature 26° C.					
	Rotation $\left(\frac{\text{degrees}}{\text{cm.}\times H}\right)$	Compensator. (Degrees.)	Retardation. (Waves.)	B_m	E_c (McComb.)	B_m/E_c	B_m Calculated. ($C=3.41\times10^{-10}$.)
440	1.64×10^{-3}	1.63	1.23×10^{-3}	10.8×10^{-13}	15.4×10^{-7}	$.70\times10^{-6}$	11.2×10^{-13}
460	1.46	1.53	1.14	10.0	13.4	.75	10.6
480	1.31	1.56	1.12	9.8	12.8	.76	9.9
500	1.16	1.51	1.10	9.6	11.8	.81	9.3
520	1.04	1.51	1.02	8.9	11.7	.76	8.8
540	.93	1.49	.96	8.4	10.5	.80	8.4
560	.85	1.48	.93	8.1	10.3	.79	7.9
580	.755	1.43	.87	7.6	9.8	.77	7.6
600	.70	1.45	.85	7.4	9.3	.79	7.3
630	.61	1.42	.79	6.9	8.6	.80	6.9
660	.56						

TABLE IX.

Mono-ethyl Aniline. Field, 13,700 *c.g.s. Temperature,* 27° *C.*

Wave-length. ($\mu\mu$)	Compensator. (Degrees.)	Retardation. (Waves.)	B_m	Refractive Index.	B_m Calculated. ($C=4.01\times10^{-10}$.)
520	1.63	1.08×10^{-3}	9.6×10^{-13}	1.5621	10.6×10^{-13}
540	1.60	1.04	9.2	1.5592	9.7
560	1.66	1.04	9.2	1.5563	9.3
580	1.68	1.02	9.0	1.5539	8.9
600	1.67	.98	8.7	1.5515	8.4
630	1.70	.95	8.4	1.5487	8.0
660	1.61	.89	7.9	1.5464	7.6

A remarkable difference in the magnitudes obtained from two different samples of brom-benzol (both from Kahlbaum) is shown in Tables XIV. and XV., the constant being over twice as large in the

TABLE X.

Di-ethyl Aniline.

Wave-length. ($\mu\mu$)	Along Field (12,900).	Across Field (13,800 c.g.s.). Temperature 25° C.					
	Rotation. $\left(\frac{\text{degrees}}{\text{cm.}\times H}\right)$	Compensator. (Degrees.)	Retardation. (Waves.)	B_m	B_e (McComb.)	B_m/B_e	B_m Calculated. ($C=2.62\times10^{-10}$.)
440	1.40×10^{-3}	1.18	$.91\times10^{-3}$	8.0×10^{-13}	15.3×10^{-7}	$.52\times10^{-6}$	7.9×10^{-13}
460	1.22	1.17	.87	7.6	13.8	.55	7.5
480	1.10	1.18	.84	7.4	13.2	.56	7.1
500	.97	1.13	.78	6.8	—	—	6.7
520	.87	1.14	.75	6.6	12.0	.55	6.3
540	.78	1.09	.70	6.1	11.2	.54	6.0
560	.69	1.03	.64	5.6	10.4	.54	5.7
580	.625	1.07	.64	5.6	9.8	.57	5.4
600	.575	1.00	.59	5.2	9.5	.55	5.2
630	.51	.95	.53	4.6	9.1	.51	4.9
660	.46	.96	.51	4.5	8.6	.52	4.7

TABLE XI.

α-mono-brom-naphthalene.

Wave-length. ($\mu\mu$)	Along Field (14,200).	Across Field (13,800 c.g.s.). Temperature 25° C.					
	Rotation. $\left(\frac{\text{degrees}}{\text{cm.}\times H}\right)$	Compensator. (Degrees.)	Retardation. (Waves.)	B_m	B_e (McComb.)	B_m/B_e	B_m Calculated. ($C=8.7\times10^{-10}$.)
440	1.80×10^{-3}	5.94	4.60×10^{-3}	40.5×10^{-13}	13.2×10^{-7}	3.06×10^{-6}	41.0×10^{-13}
460	1.57	5.80	4.35	38.0	12.4	3.05	38.0
480	1.42	5.68	4.10	36.0	11.5	3.13	35.8
500	1.27	5.42	3.80	33.4	11.0	3.03	33.7
520	1.14	5.44	3.65	32.0	10.2	3.14	31.8
540	1.03	5.31	3.45	30.3	9.8	3.09	30.1
560	.94	5.23	3.30	29.0	9.5	3.05	28.6
580	.86	5.10	3.10	27.2	9.0	3.02	27.5
600	.79	5.14	3.00	26.3	8.7	3.02	26.3
630	.71	4.96	2.80	24.5	7.9	3.10	24.5
660	.66	5.01	2.70	23.7			23.1

TABLE XII.

Chlor-Benzol. (*Sample No. 1.*) *Field,* 13,800 *c.g.s. Temperature* 25° C.

Wave-length. ($\mu\mu$)	Compensator. (Degrees.)	Retardation. (Waves.)	B_m	B_e (McComb.)	B_m/B_e
440	1.81	1.39×10^{-3}	12.2×10^{-13}	19.5×10^{-7}	$.65\times10^{-6}$
460	1.85	1.38	12.1	17.8	.68
480	1.87	1.34	11.8	16.5	.71
500	1.76	1.22	10.7	15.6	.69
520	1.67	1.12	9.8	15.0	.65
540	1.67	1.08	9.5	14.1	.67
560	1.54	.96	8.4	13.9	.60
580	1.62	.98	8.6	12.9	.66
600	1.59	.93	8.2	12.3	.67
630	1.66	.93	8.2	11.4	.72
660	1.56	.85	7.5	10.8	.69

TABLE XIII.

Chlor-Benzol. (*Sample No.* 2.)

Wave-length. ($\mu\mu$)	Along Field (14,200).	Across Field (13,200 c.g.s.). Temperature 25° C.				
	Rotation. $\left(\frac{\text{degrees}}{\text{cm.}\times H}\right)$	Compensator. (Degrees.)	Retardation. (Waves.)	B_m	B_e (McComb.)	B_m/B_e
440	$.95\times10^{-3}$	2.00	1.54×10^{-3}	13.8×10^{-13}	13.2×10^{-7}	1.04×10^{-6}
460	.85	2.00	1.49	13.4	12.6	1.06
480	.78	1.95	1.39	12.5	11.6	1.08
500	.70	1.85	1.28	11.5	11.2	1.03
520	.64	1.80	1.19	10.7	10.2	1.05
540	.58	1.87	1.21	10.9	9.9	1.10
560	.52	1.91	1.19	10.7	10.2	1.05
580	.50	1.85	1.12	10.1	9.8	1.03
600	.46	1.90	1.13	10.2	8.3	1.22
630	.42	1.88	1.05	9.5	8.6	1.10
660	.38	1.79	1.02	9.2	8.3	1.11

one as in the other. A test by McComb of the same samples showed no correspondingly large difference in the electric field. Two samples of chlor-benzol were also tested (Tables XII. and XIII.) which showed an appreciable difference in magnitude of B_m though not so large as brom-benzol. These two liquids were found to be unsatisfactory for making optical investigations owing to the field of view being periodically disturbed apparently by some motions within the liquid. The other liquids investigated did not reveal any appreciable variations in the constant with different samples.

TABLE XIV.

Brom-Benzol. (*Sample No.* 1.) *Field,* 13,800 *c.g.s. Temperature,* 25° *C.*

Wave-length. ($\mu\mu$)	Compensator. (Degrees.)	Retardation. (Waves.)	B_m
440	1.80	1.39×10^{-3}	12.2×10^{13}
460	1.82	1.36	11.9
480	1.80	1.30	11.4
500	1.71	1.19	10.4
520	1.69	1.13	9.9
540	1.72	1.12	9.8
560	1.64	1.04	9.1
580	1.70	1.04	9.1
600	1.70	1.00	8.8
630	1.74	.98	8.6
660	1.72	.93	8.2

TABLE XV.

Brom-Benzol. (*Sample No.* 2.)

Wave-length. ($\mu\mu$)	Along Field (14,200).	Across Field (13,600 c.g.s.). Temperature 25° C.				
	Rotation. $\left(\frac{\text{degrees}}{\text{cm.}\times H}\right)$	Compensator. (Degrees.)	Retardation. (Waves.)	B_m	B_e (McComb.)	B_m/B_e
440	1.04×10^{-3}	4.00	3.06×10^{-3}	27.0×10^{13}	14.0×10^{7}	1.93×10^{-6}
460	.95	4.02	2.99	26.3	13.4	1.96
480	.86	4.03	2.90	25.5	10.9	2.33
500	.78	4.10	2.85	25.0	11.3	2.20
520	.71	4.20	2.78	24.5	10.4	2.15
540	.64	4.14	2.68	23.6	10.0	2.36
560	.60	4.10	2.60	22.9	9.9	2.30
580	.55	4.07	2.50	22.0	8.1	2.70
600	.51	4.09	2.41	21.2	8.8	2.30
630	.46					
660	.43					

Carbon bisulphide (Table XVI.) was the only liquid tested in which the constant in the magnetic field was negative to that in the electric.

An inspection of the tables reveals that the ratio B_m/B_e is, for those liquids giving reliable results, practically constant throughout the spectrum, the fluctuation in the value being such as would be ascribed to experimental errors. The values of B_m as calculated from Havelock's formula agree equally well with the measured quantities. There seems to be a general tendency of the calculated

TABLE XVI.

Carbon Bisulphide.

Wave-length. ($\mu\mu$)	Along Field (13,200).	Across Field (13,800 c.g.s.). Temperature 28° C.					
	Rotation. $\left(\frac{\text{degrees}}{\text{cm.}\times H}\right)$	Compensator. (Degree.)	Retardation. (Waves.)	B_m	B_e (McComb.)	B_m/B_e	B_m Calculated. ($C=1.44\times10^{-10}$).
440	1.38×10^{-3}	−.95	$-.74\times10^{-3}$	-6.5×10^{-13}	4.17×10^{-7}	-1.56×10^{-6}	-6.2×10^{-13}
460	1.23	—	—	—	—	—	—
480	1.13	−.90	−.65	−5.7	3.91	−1.46	−5.4
500	1.03	−.86	−.60	−5.7	3.62	−1.47	−5.1
520	.95	−.79	−.53	−4.7	3.43	−1.37	−4.7
540	.86	−.75	−.49	−4.3	3.23	−1.33	−4.5
560	.79	−.81	−.51	−4.5	3.07	−1.46	−4.3
580	.73	−.74	−.46	−4.0	2.90	−1.38	−4.1
600	.67	−.70	−.42	−3.7	2.79	−1.33	−3.9
630	.60	−.68	−.39	−3.4	2.63	−1.31	−3.6
660	.55	−.73	−.40	−3.5	2.45	−1.43	−3.4

TABLE XVII.

Toluidin. Length of Cell, 4 cm.

Wave-length. ($\mu\mu$)	Along Field (14,200).	Across Field (13,800 c.g.s.).				
	Rotation. $\left(\frac{\text{degrees}}{\text{cm.}\times H}\right)$	Compensator. (Degrees.)	Retardation. (Waves.)	B_m	Refractive Index.	B_m Calculated. ($C=2.39\times10^{-10}$.)
440	1.31×10^{-3}					
460	1.17					
480	1.06					
500	.94	.71	$.50\times10^{-3}$	6.6×10^{-13}	1.5814	6.8×10^{-13}
520	.85	.73	.49	6.4	1.5778	6.4
540	.77	.73	.48	6.3	1.5746	6.1
560	.71	.70	.44	5.8	1.5721	5.8
580	.64	.67	.42	5.5	1.5696	5.6
600	.59	.69	.41	5.4	1.5670	5.4
630	.54	.69	.40	5.3	1.5645	5.1
660	.50					

value at 440 $\mu\mu$ to run above the observed; but since the ratio B_m/B_e for this wave-length is in some cases lower, in others higher than the rest, we are inclined to ascribe the divergence to the difficulty of making accurate observations in this region of the spectrum.

THE BRACE LABORATORY OF PHYSICS,
UNIVERSITY OF NEBRASKA.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE FORTY-SEVENTH MEETING.

A REGULAR meeting of the Physical Society was held in the Palmer Laboratory of Princeton University on Saturday, October 23, 1909. President Henry Crew presided.

The following papers were presented:

1. The Relationship Between Entropy and Time. W. S. FRANKLIN.

2. The Lorentz Shortening: an Apparent Paradox. GILBERT N. LEWIS.

3. Ionization Produced by Entladungsstrahlen and Experiments on the Nature of the Radiation. ELIZABETH R. LAIRD.

4. The Effect of NO_3 and Al on the Uranyl Bands and of Ca and Temperature on Neodymium Bands. W. W. STRONG.

5. The Octave Overtone from Tuning Forks. D. C. MILLER.

6. The Electrostatic Effect of a Changing Magnetic Field. J. M. KUEHNE.

7. A Modification of the Thomson-Wilson Method of Determining the Elementary Electrical Charge, and the Most Probable Value of that Charge. R. A. MILLIKAN.

8. Polarization of Röntgen Rays. WM. R. HAM.

9. A New Radiant Emission from the Spark. R. W. WOOD.

10. The Dependence of the Photo-electric Current on the Wave-length of Incident Light. F. K. RICHTMYER.

11. Kinetic Energy of Thermions. O. W. RICHARDSON.

12. Contact Difference of Potential in the Magnetic Field. E. P. ADAMS.

13. A Neglected Form of Relativity. D. F. COMSTOCK.

14. The Relation between the Velocity of Light and the Velocity of its Source. R. C. TOLMAN.

15. The Radiation from Platinum. (By title.) A. TROWBRIDGE.

16. A Characteristic of Spectral Energy Curves. (By title.) W. W. COBLENTZ.

17. The Depth of Complete Scattering of Kathode Rays in Lead and the Variation of the Depth with Velocity of the Rays. WM. R. HAM.

18. Landscapes and Still Life in Infra-red and Ultra-violet. R. W. WOOD.

19. Some Laboratory Applications of the Photo-electric Current. F. K. RICHTMYER.

20. Reflection of Slow-Speed Electrons. O. W. RICHARDSON.

21. The Temperature Radiation of Transparent Bodies. R. W. WOOD.

22. A Thermo-luminescent Glass. W. W. COBLENTZ.

ERNEST MERRITT,
Secretary.

A CHARACTERISTIC OF SPECTRAL ENERGY CURVES.[1]

BY W. W. COBLENTZ.

USING the Wien spectral energy equation it can be shown that the wave-length of maximum emission can be computed from the observed spectral energy curve by taking two wave-lengths, λ_1 and λ_2, corresponding to any two points where the emissivity $E_1 = E_2$. Paschen observed this property before the establishment of the theoretical equation. In the subsequent work of Paschen, and of Lummer and Pringsheim, this point is passed over without discussion. In fact the latter observers do not (to my knowledge) say how they obtain the point of maximum emission from the spectral energy curve.

From the beginning of my work on this subject it was observed that in computing the λ_{max} by the aforesaid method the value usually decreases uniformly from a high one (several per cent. higher) when λ_1 and λ_2 are far apart, to a fairly constant value for λ_1 and λ_2 taken close together. In the spectra of metals the λ_{max} was therefore computed by taking values of λ_1, and λ_2 close together, assuming that the discrepancy was due to experimental error. In looking over previous work by Paschen it was found that his data indicate a similar condition. Unfortunately Lummer and Pringsheim never published a full report of their observations.

From the complete solution of Planck's equation

$$\lambda_{max} = \frac{(\log \lambda_2 - \log \lambda_1)\lambda_1\lambda_2}{(\lambda_2 - \lambda_1)\log \varepsilon} - \left(\frac{\log\left[\left(e^{\frac{a\lambda_m}{\lambda_1}} - 1\right)e^{\frac{\lambda_m}{\lambda_2}}\right] - \log\left[\left(e^{\frac{a\lambda_m}{\lambda_2}} - 1\right)e^{\frac{a\lambda_m}{\lambda_1}}\right]}{\frac{a(\lambda_2 - \lambda_1)\log \varepsilon}{\lambda_1\lambda_2}} \right).$$

The first member on the right hand side of the equation is the usual solution for λ_{max}, from Wien's equation, while the second member enters as a correction factor. This correction is never negligible even when

[1] Abstract of a paper presented at the Princeton meeting of the Physical Society, October 23, 1909.

$\lambda_1 = \lambda_2 = \lambda_{max}$ (data by extrapolation, by taking λ_1 close to λ_{max}), where it is a minimum. This minimum correction to λ_{max} amounts to 3.1 per cent. taken at a temperature of 465° C., and decreases to 1.2 per cent. at 1430° C.

From this it appears that, for a black body, $\lambda_{max}\, T =$ const. should not be found constant for all temperatures, as heretofore observed, before applying this correction.

POLARIZATION OF RÖNTGEN RAYS.[1]

BY WM. R. HAM.

A DETERMINATION, by means of electroscopes, of the relative intensity of Röntgen rays in various directions from the target, using tubes especially constructed to prevent error due to absorption of glass walls, absorption in target, and secondary rays. The results show a change indicating a polarization of the order of magnitude found by Barkla by indirect methods. The effect on this polarization of absorbing sheets of various materials is such as can be accounted for by assuming the Röntgen ray beams to be made up of rays of two types, one showing very little, if any, polarization.

DEPTH OF COMPLETE SCATTERING OF KATHODE RAYS IN LEAD AND VARIATION OF THE DEPTH WITH VELOCITY OF THE RAYS.[1]

BY WM. R. HAM.

SINCE Röntgen rays originate at a finite depth in the target, there is a certain amount of absorption in the target that varies in different directions. With an adjustable target this variation together with data concerning the coefficient of absorption for the material of the target for rays of the hardness under consideration, make possible the determination of the mean depth at which the Röntgen rays originate. With a sufficiently high velocity, the kathode rays are completely scattered at this depth as is known from the fact that the Röntgen rays then show practically no polarization. This depth of complete scattering is found to vary directly with the potential across the tube. At the velocity of beta rays of uranium, the depth in lead is found to be of the same order of magnitude as that found by Crowther for gold.

[1] Abstract of a paper presented at the Princeton meeting of the Physical Society, October 23, 1909.

The Effect of NO_3 and Al on the Uranyl Bands and of Ca and Temperature on Neodymium Bands.[1]

By W. W. Strong.

A CONTINUATION of the work upon absorption spectra has brought out some new phenomena. The absorption spectra of uranyl salts consist of twelve bands in the blue and violet. These bands are about 50 Ångström units wide, have hazy edges and are about 100 Ångström units apart. Starting from the blue end of the series the bands will be designated by the letters *a*, *b*, *c*, etc. In water solutions it is found that the bands of uranyl nitrate have shorter wave-lengths than the bands of the other salts. The following table gives the approximate wave-lengths of the uranyl bands for aqueous solutions.

Uranyl Bands.

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*
Nitrate.	4870	4705	4550	4390	—	4155	4030	3905	3815	3710	3600
Bromide.	4880	4720	4560	4450	4280	4160	—	—	—	—	—
Sulphate.	4900	4740	4580	4460	4332	4200	4070	3970	3850	3740	—
Acetate.	4910	4740	4595	4455	4310	4160	4070	3970	3865	—	—
Chloride.	4920	4740	4560	4460	4315	4170	4020				

Some of the bands appear especially diffuse (chloride for example) and can only be measured approximately. The *a* and *b* bands come out best in the photographs.

Uranous salts are green and have an entirely different absorption spectra compared with that of the uranyl salts. Following are the wave-lengths of a few of the uranous bands for aqueous solutions.

Uranous Bands.

Nitrate.	6700	6470	—
Sulphate.	6730	6550	5500
Chloride.	6730	6530	5510

Becquerel (Commun. Phys. Lab. Univ. Leiden, Nos. 110, 111) gives the following wave-lengths of the phosphorescent bands of some uranyl salts at a temperatures of 80°(K.).

In the case of the phosphorescent bands there are several series of bands. From these three tables it seems that the NO_3 group possesses an

[1] Abstract of a paper presented at the Princeton meeting of the Physical Society, October 23, 1909.

influence upon the frequency of vibration of the absorbing or emitting vibrators of the uranium compound. If it were possible to find the value of e/m for these vibrators it might be possible to find the force exerted by the NO_3 group upon each vibrator. It is quite remarkable that within widely different concentrations the wave-length of the uranyl bands do not change. Probably only a small percentage of the NO_3 groups take part in a light absorption or emission and these may possess properties that are quite different from those of the greater number of NO_3 groups.

Uranyl Phosphorescent Bands.

Nitrate.	—	5069.6	5301.5	5554.6	5832.9	—
Sulphate.	4918.3	5133.9	5369.4	5626.8	5910.1	6219.5
Double acetate, uranyl and sodium.	4932.5	5148.6	5384.5	5642.7	—	—

The addition of sufficient aluminium chloride to a water solution of uranyl chloride or of calcium chloride to a methyl alcohol solution of uranyl chloride is found to cause the *d* and *e* bands to come together, so as to form a single wide band and to cause the other uranyl bands to change their positions so that the resulting series of bands is almost the same as that of uranyl chloride in ethyl alcohol The general effect of the presence of calcium and aluminium is to shift the first bands (*a*, *b*, etc.) of the series towards the red.

Rise of temperature (0° to 90° C.) causes very little if any change in the wave-length of the neodymium bands. The bands at the higher temperatures are invariably broader. When calcium chloride is present many of the neodymium bands narrow with rise of temperature and are shifted towards the red. It may be possible that many of the temperature and Zeeman effects found by Becquerel are due to the presence of foreign bodies in the neodymium and erbium compounds with which he worked. It is also possible that the presence of calcium in the sun might alter the Zeeman effect so that the strength of field required to produce a given separation would be different from that required in the laboratory where only pure substances are used.

THE KINETIC ENERGY OF THE THERMIONS.[1]

BY O. W. RICHARDSON.

THE present investigation is a continuation of one published in the *Philosophical Magazine* (Dec., 1908) under the title of "The Kinetic Energy of the Ions Emitted by Hot Bodies." The apparatus

[1] Abstract of a paper presented at the Princeton meeting of the Physical Society, October 23, 1909.

used involves the same principles as that already employed but it has been greatly improved in detail. The increased accuracy of measurement and the greater extent to which the theoretical conditions are satisfied enable the author's previous conclusions to be subjected to a more rigorous test. In particular the spreading out of the negative electrons emitted by a narrow strip in the absence of an electric field has been examined and is found to agree very satisfactorily with the distribution which was predicted in the earlier paper, on the assumption that the distribution of velocity among the electrons is in accordance with Maxwell's law.

It is found however that at points at a considerable distance from the center of the system the current through the slit is in excess of the theoretical requirements. The discrepancy is found to be much greater when an electric field is applied between the two plates than when the field is absent. It appears to be due to the reflection of the electrons at the surface of the receiving plates. This difficulty does not appear to arise when the "sideways kinetic energy" of the positive ions is being measured.

The Reflection of Low-Speed Electrons by Metal Surfaces.[1]

By O. W. Richardson.

In the author's investigations of the properties of the electrons emitted by hot bodies phenomena have frequently arisen which have only seemed capable of explanation on the view that the slow-speed electrons dealt with gave rise to a reflected or secondary radiation when they struck a metal surface.[2] The present communication describes an experiment in which the result of this reflection is exhibited using electrons whose speed corresponds to a fall of potential of 8 volts. Indirect evidence is also adduced in favor of the view that the unaccelerated electrons also give rise to a similar effect. Their speed corresponds to a fall of potential of about .03 volt. It appears that this reflection has previously been observed by von Baeyer[3] who concluded that the effect occurred with speeds corresponding to 5 volts and upwards.

[1] Abstract of a paper presented at the Princeton meeting of the Physical Society, October 23, 1909.

[2] See for example Phil. Mag. [6], vol. 16, 761, 1908, and later papers in the same journal.

[3] Ber. Deutsch. Phys. Gesell., 10, 4, p. 96, 1908.

ON THE ELECTROSTATIC EFFECT OF A CHANGING MAGNETIC FIELD.[1]

BY J. M. KUEHNE.

A CHARGED body suspended in a changing magnetic field, or better in a field swept by magnetic lines of force, should, according to the electro-magnetic theory of Maxwell and others, be subjected to a force at right angles to the lines of magnetic force as well as to the direction of their motion. The magnitude of this force will depend on the product $Q \cdot dN/dt$, where Q is the quantity of the charge and dN/dt the rate at which magnetic lines are moving across the space occupied by the charge.

The apparatus used in the experiment consists of a parallel plate condenser having twenty pairs of ring-shaped plates — the one set of plates is suspended free to rotate, the oppositely charged plates are stationary. Through the centers of both sets of plates passes one limb of the closed magnetic circuit of a transformer. The magnetic circuit is excited by an alternating current, and the condenser is charged from the secondary of a potential transformer, whose primary is connected in parallel with the magnetic circuit. The charged plates are thus subjected to a torque whose value, according to theory, is $L = \frac{1}{2}nNCE$, where n is the frequency of the alternating magnetic field, N the maximum magnetic flux through the transformer core, C the capacity of the condenser, and E the maximum charging e.m.f.

E_1	E_2	L	L'
600	.345	.33	.26
606	.342	.33	.35
606	.490	.47	.45
600	.492	.47	.44
956	.345	.52	.48
956	.348	.52	.46
806	.444	.56	.53
772	.485	.59	.57
828	.490	.64	.63
840	.494	.65	.64
848	.495	.66	.67
864	.485	.66	.73
916	.490	.71	.66
956	.474	.72	.70
956	.490	.74	.72
956	.492	.74	.72
1,294	.480	.98	1.05
1,314	.498	1.03	1.04

[1] Abstract of a paper presented at the Princeton meeting of the Physical Society October 23, 1909.

In the experiment N was varied between 127,000 and 190,000 lines, and E between 850 and 1,860 volts, the other factors remaining constant. The effect was thus tested over a range of variation of more than 3 : 1, and throughout this range the observed torque was in satisfactory agreement with that demanded by theory, both in direction and magnitude. The observed torque was measured by rotation of the torsion head through a known angle, the moment of torsion of the suspending fiber being known.

In the following table of results E_1 indicates the "effective" charging E.M.F., E_2 the effective E.M.F. induced by the magnetic field in each turn of a coil surrounding the magnet core, L the torque in dyne cm. calculated from theory, and L' the observed torque.

Ionization Produced by Entladungsstrahlen and Experiments on the Nature of the Radiation.[1]

By Elizabeth R. Laird.

THIS work is a continuation of that previously reported. Experiments have been made to show that the effect of Entladungsstrahlen cannot be due to dust driven from the electrodes, nor to electrostatic induction, nor to a mechanical pressure wave, nor to light of wave-length greater than 1,000.

Other experiments show that the same radiation ionizes the gas through which it passes, since the ionization is increased in proportion to the thermoluminescence, and is transmitted or not by substances according as they transmit or cut off thermoluminescence, and remains unchanged under conditions in which the thermoluminescence is unchanged. In particular the ionization is cut off by thin quartz. It is much increased by the presence of certain vapors, notably turpentine, ammonia, ether, methyl iodide. The radiation appears to be reflected very slightly, but is reflected in about the same degree by such various substances as paper, lead, copper, glass. Some experiments seem to show that the penetrating power of the radiation differs according to the source used. A fuller report will appear shortly in this journal.

Mount Holyoke College.

[1] Abstract of a paper presented at the Princeton meeting of the Physical Society, October 23, 1909.

A NEW MODIFICATION OF THE CLOUD METHOD OF MEASURING THE ELEMENTARY ELECTRICAL CHARGE, AND THE MOST PROBABLE VALUE OF THAT CHARGE.[1]

BY R. A. MILLIKAN.

THIS modification of the cloud method of determining e consists:

1. In making observations, not upon the surface of a cloud, but upon single isolated drops carrying multiple charges.

2. In exactly balancing gravity upon these single charged drops by an electrical field.

3. In observing the rate of fall of these same drops under gravity after the electrical field has been thrown off.

4. In eliminating any possible error due to evaporation by first obtaining stationary, *i. e.*, balanced, drops, and then measuring the times of passage of these same drops across equal spaces in the field of the reading telescope.

5. In directly measuring the temperature of the cloud chamber instead of computing it. (It is this computation which was found to have introduced the chief error into preceding determinations of e by the cloud method.[2]

In obtaining the results herewith reported the distances through which the falling drops were timed varied from 1 mm. to 1.55 mm.; the times of fall through these spaces from 3 to 5 seconds: the distances between the plates to which the potentials were applied from 4.63 to 5.45 mm.: the intensities of the electrical fields from 14 to 17 absolute electrostatic units: the liquids used for forming drops were alcohol and water.

The method compares favorably in directness and precision with any which has thus far been used for determining e. The results of 7 very concordant observations on water drops carrying triple positive charges gave $e = 4.59 \times 10^{-10}$: of 11 concordant observations on alcohol drops carrying double positive charges gave $e = 4.64 \times 10^{-10}$: of 10 observations on water drops carrying quadruple positive charges gave $e = 4.56 \times 10^{-10}$: of 5 observations on water drops carrying quintuple positive charges gave $e = 4.83 \times 10^{-10}$: of three observations on water drops carrying sextuple positive charges gave $e = 4.69 \times 10^{-10}$: of two observations on water drops carrying double positive charges gave $e = 4.87 \times 10^{-10}$. The weighted mean of these results is 4.65×10^{-10}. The error in this determination is estimated as not more than 2 per cent. Mr. Begeman has just completed in this laboratory a very long and careful series of observations by the regular Wilson method timing however the layers corresponding to multiple charges, and obtains as his final mean $e = 4.66 \times 10^{-10}$.

[1] Abstract of a paper presented at the Princeton meeting of the Physical Society, October 23, 1909.

[2] Millikan and Begeman, PHYS. REV., 26, p. 197, 1908.

The mean of all the recent determinations of e by methods which seem least open to question is given below.

Planck	4.69
Rutherford and Geiger	4.65
Regener	4.79
Millikan	4.65
Begeman	4.66

Mean $e = 4.69 \times 10^{-10}$

A Thermoluminescent Glass.[1]

By Wm. W. Coblentz.

An experimental demonstration of a thermoluminescent glass made of feldspar, formed into a perfectly transparent rod which was heated electrically. The luminiscence appears as a faint white cloud, at a temperature of about 800° C., which increases in intensity with rise in temperature. The important point illustrated was that the light is white at all temperatures, *i. e.*, it does not pass through the usual chromatic changes found in the radiation from opaque substances which would appear red at these temperatures. The absorption and behavior in a magnetic field will require further investigation.

The Dependence of Photo-Electric Current on Wave-Length of Incident Light.[1]

By F. K. Richtmyer.

Using the apparatus previously described [2] the author has investigated the relation between wave-length of the incident light and the resulting photo-electric current from a sensitive sodium surface. The spectrum of a Nernst glower was thrown on the sensitive surface by means of a spectrometer with a slit substituted for the eyepiece and the resulting photo-electric currents measured by the rate of drift of an electrometer from $\lambda = .65\,\mu$ to $\lambda = .42\,\mu$.

More satisfactory results were obtained by use of the spectrum of acetylene. In this case the observed currents were corrected for (1) variable dispersion of the prism and (2) energy distribution in the spectrum of acetylene. The curve plotted between corrected photo-electric currents and wave-lengths is similar to the visibility curve but has a maximum at $\lambda = .46\,\mu$, suggesting resonance in some form with the free period of the electrons in the atom of sodium in its solid state corresponding to the period of $\lambda = .46\,\mu$.

[1] Abstract of a paper presented at the Princeton meeting of the Physical Society, October 23, 1909.

[2] Phys. Rev., Vol. XXIX., pp. 71–80.

Computation of the energy changes involved shows that, in a certain case, while approximately 100 ergs per second of energy were absorbed from the incident light, the energy of the expelled electrons was less than one erg per second.

LABORATORY APPLICATIONS OF THE PHOTO-ELECTRIC CURRENT.[1]

BY F. K. RICHTMYER.

THE production of measurable photo-electric currents from a sodium surface by extremely feeble illuminations recommends this phenomenon for use in making photometric measurements of monochromatic or of isochromatic light sources where observations by eye are extremely difficult. For example the author has studied the decay of phosphorescence of a sample of Balmain's paint for a period of over one hour following a half-minute excitation by sunlight. The curve is of the usual form and when plotted with $1/\sqrt{I}$ as ordinates, shows three distinct linear branches.

It is also pointed out that this combination of photo-electric cell and electrometer acts as a time integrator for light intensity in much the same way that an electrolytic cell would measure total quantity of electricity. In this connection the cell might be used to study short decay phosphorescence or, as has been done by the author, to measure the *total* quantity of light passing through a photographic shutter on exposure.

[1] Abstract of a paper presented at the Princeton meeting of the Physical Society, October 23, 1909.

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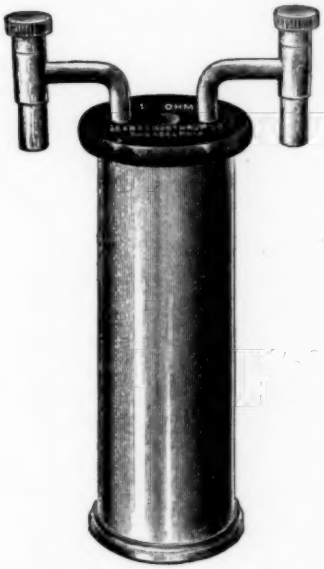
ERRATA.

Page 286, the upper figure is Fig. 7; the lower figure is Fig. 6.

Page 325, the left side of the second equation should read pF instead of F.

Page 326, eighth line from bottom, interchange 1 and c_1/c_2.

N.B.S.
RESISTANCE STANDARD



The new form of Resistance Standard designed by Dr. Rosa of the National Bureau of Standards.

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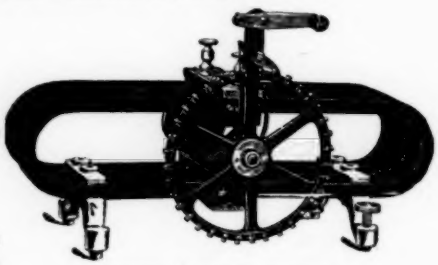

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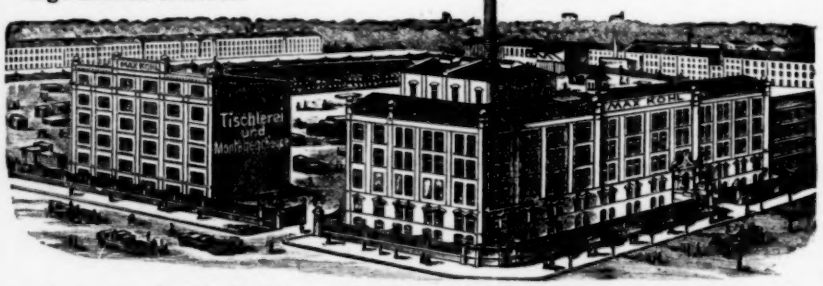

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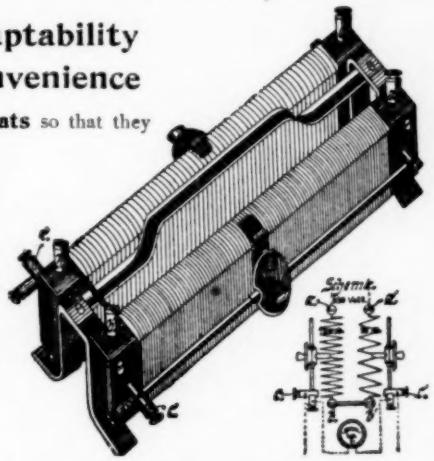

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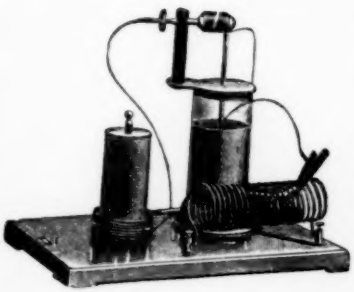

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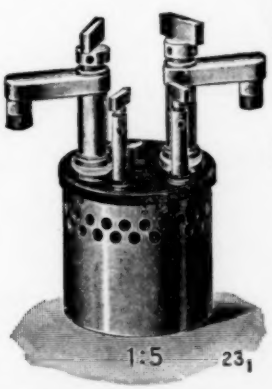

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